**Pop-ups (Grades 4-5)**

Teacher’s Guide

Overview

This unit introduces paper pop-up mechanisms as a basis for learning elementary science, engineering and math. Pop-ups are three dimensional linkages, which can be made from heavy paper or card stock. Most students have already seen pop-up books and cards, are intrigued by them and would like to make their own. These materials develop spatial visualization, measurement of angles and distances, data analysis, concepts of symmetry and motion, systems thinking, design and troubleshooting.

Common Core Learning Standards for ELA

Common Core Writing Standards 4-5

**Text Types and purposes**

2. Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization and analysis of content.

**Production and Distribution of writing**

4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

5. With guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising and editing.

**Research to Build and Present Knowledge**

7. Conduct short research projects that build knowledge through investigation of different aspects of a topic.

Common Core Speaking and Listening Standards 4-5

**Comprehension and Collaboration**

1. Engage effectively in a range of collaborative discussions with diverse partners, building on others’ ideas and expressing their own clearly.

**Presentation of Knowledge and Ideas**

4. Report on a topic, sequencing ideas logically and using appropriate facts and relevant, descriptive details to support main ideas or themes.

Common Core Language Standards 4-5

**Vocabulary acquisition and use**

4. Demonstrate or clarify the meaning of unknown or multiple-meaning words and phrases, choosing flexibly from a range of strategies.

6. Acquire and use accurately a range of general academic and domain-specific words and phrases.

Common Core Learning Standards for Mathematics

Standards for Mathematical Practice

**MP1. Make sense of problems and persevere in solving them:** Understand the meaning of a problem and look for entry points to its solution. Analyze givens, constraints, relationships and goals. Make conjectures about the form and meaning of the solution and plan a solution pathway.

**MP2. Reason abstractly and quantitatively:** Make sense of quantities and their relationships in problem situations. Represent a given situation symbolically and manipulate the representing symbols, pausing as needed to probe into the referents for the symbols.

**MP3. Construct viable arguments and critique the reasoning of others:** Understand and use stated assumptions, definitions, and previously established results in constructing arguments. Reason inductively about data, making plausible arguments that take into account the context from which the data arose.

**MP4. Model with mathematics:** Identify important quantities in a practical situation, and map and analyze their relationships mathematically. Interpret mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

**MP 6. Attend to precision:** Communicate precisely to others, using clear definitions of concepts and symbols and carefully formulated explanations.

**MP 7.** **Look for and make use of structure**: Look closely to discern a pattern or structure.

Standards for Mathematical Content (Grades 4 & 5)

**4.OA & 5.OA Operations and Algebraic Thinking**4.OA5: Generate and analyze patterns  
5.OA3: Analyze patterns and relationships

**4.MD Measurement and Data**4.MD4: Represent and interpret data  
4.MD5, 6 & 7: Geometric measurement: understand concepts of angle and measure angles

**4.G & 5.G Geometry**4.G1, 2 & 3: Draw and identify lines and angles, and classify shapes by properties of their lines and angles  
5.G3 & 4: Classify two-dimensional figures into categories based on their properties

Next Generation Science Standards/ Frameworks for K-12 Science Education

Dimension 1: Scientific and Engineering Practices:

1. **Asking questions and defining problems:** Students should be able to ask questions of each other about the phenomena they observe and the conclusions they draw from their models or scientific investigations. For engineering, they should ask questions to define the problem to be solved and to elicit ideas that lead to the constraints and specifications for its solution.
2. **Developing and using models:** Students should be asked to use diagrams, maps and other abstract models to as tools that enable them to elaborate on their own ideas, develop explanations and present them to others.
3. **Planning and carrying out investigations:** In the elementary years, students’ experiences should be structured to help them learn to define the features to be investigated, such as patterns that suggest causal relationships.
4. **Analyzing and interpreting data:** At the elementary level, students need support to recognize the need to record observations – whether in drawings, words or numbers – and to share them with others.

**6. Constructing explanations and designing solutions:** The process of developing a design is iterative and systematic, as is the process of developing an explanation in science. Elements that are distinctive in engineering include specifying constraints and criteria for desired qualities of the solution, developing a design plan, producing or testing models or prototypes, selecting among alternative design features, and refining design ideas based on the performance of a prototype.

**7. Engaging in argument from evidence:** In engineering, reasoning and argument are essential to finding the best possible solution to a problem. At an early design stage, competing ideas must be compared (and possibly combined), and the choices are made through argumentation about the merits of the various ideas pertinent to the design goals.

**8. Obtaining, evaluating and communicating information:** In engineering,. Students need opportunities to communicate ideas using appropriate combinations of sketches, models and language. They should also create drawings to test concepts and communicate detailed plans; explain and critique models, and present both planning stages and ultimate solutions.

Dimension 2: Crosscutting concepts:

1. **Patterns:** Noticing patterns is often a first step to organizing and asking scientific questions about why and how the patterns occur. In engineering, it is important to observe and analyze patterns of failure in order to improve a design.
2. **Cause and effect: mechanism and prediction:** Any application of science, or any engineered solution to a problem, is dependent on understanding the cause-and-effect relationships between events. The process of design is a good place to start, because students must understand the underlying causal relationships in order to devise and explain a design to achieve a specified objective.
3. **Scale, proportion and quantity:** The concept of scale builds from the early grades as an essential element of understanding phenomena. Young children can begin understanding scale with objects, space and time related to their world and with scale models and maps.
4. **Systems and system models:** A system is an organized group of related objects or components that form a whole. Models can be valuable in predicting a system’s behaviors or in diagnosing its problems and failures. Students express their thinking with drawings or diagrams and with written or oral descriptions. They should describe objects in terms of their parts and the role those parts play in the functioning of the object.

**6. Structure and function:** The functioning of systems depends on the shapes and relationships of certain key parts, as well as on the properties of the materials. Exploration of the relationship between structure and function can begin in the early grades through investigations of accessible systems in the natural and human-built world.

**7. Stability and change:** Much of science and mathematics has to do with understanding how change occurs in nature and in social and technological systems, and much of engineering has to do with creating and controlling change.

Dimension 3: Disciplinary Core Ideas – Physical Science:

**Core Idea PS2: Motion and Stability: Forces and Interactions**Interactions between any two objects can cause changes in one or both of them. An understanding of the forces between objects is important for describing how their motions change, as well as for predicting stability or instability in systems.

Dimension 3: Disciplinary Core Ideas – Engineering, Technology and Applications of Science

**Core Idea ETS1: Engineering Design**Identification of a problem and the specification of clear design goals, contending with constraints, using models to better understand the features of a design problem, compare designs, test them and compare their strengths and weaknesses. Selection of a design often requires making trade-offs among competing criteria.

Curriculum Map

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Lesson** | **Title** | **Summary** | **Approx. time (min.)** | **Vocabulary** | **Extensions** | **Assessment Methods** |
| 1 | **Looking Closely at Pop-ups** | Examining commercially made pop-ups to learn about their structure | 100 | Simple, complex, attachment, book, force, mechanism, pop-up |  | Observation, discussion, written work |
| 2 | **Make a  Pop-up** | Students try to create and troubleshoot their own pop-ups | 100 | troubleshooting, hinge, link, linkage, gutter, page position, fold, paper engineer, issue, cause, fix |  | Observation, discussion, written work |
| 3 | **The Shape  of a Pop-up** | Exploring how the shapes change when a pop-up is closed and opened for the first time | 100 | Score, burnish, prediction, test, results, point of view, edge view, arc, triangle, experiment |  | Observation, discussion drawings |
| 4 | **Finding Patterns in the Folds** | Looking for how and where a pop-up makes a fold | 100 | asymmetry, controlled variable, height, independent variable, input variable, dependent variable, symmetry, variable, output variable, controlled experiment | Changing the variables | Observation, discussion, written work |
| 5 | **The Pop-up Computer** | Measuring fold patterns and looking for patterns in the data | 150 | measurement, length, hinge, link, data collection, data table, data analysis, pattern, hypothesis, equation, symbolic representation | 1. How can you make it lie flat?  2. What happens if you break the rule? | Observation, student projects |
| 6 | **Where does it Hide?** | Analyzing and explaining the patterns in the fold positions | 50 | Cause, effect, explanation, opaque, transparent, point of view, top view, edge view, rhombus, quadrilateral, rectangle, square, system of simultaneous equations | 1. How can you make it stay inside the book?  2. Geometry through pop-ups | Observation, discussion, written work |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Lesson** | **Title** | **Summary** | **Approx. time (min.)** | **Vocabulary** | **Extensions** | **Assessment Methods** |
| 7 | **Stair-step  Pop-ups** | Making compound pop-ups that combine simple pop-ups | 100 | system, system diagram, compound system, subsystem, fraction, input, output, parallel connection, series connection, acute angle, right angle, obtuse angle, similar shapes | 1. Add more pop-ups  2. System diagrams for more complex pop-ups | Observation, discussion, written work |
| 8 | **How they Move** | Exploring the directions of motion of different kinds of pop-ups | 100 | angle-fold pop-up, axis, dimension, two-dimensional, plane, space, three-dimensional, direction of motion, flag, horizontal, vertical, force component, intersecting lines, parallel lines, parallel-fold pop-up, perpendicular, skew lines, vertex | A pop-up whose hinges are skew | Observation, discussion, written work |
| 9 | **Make an Angle-Fold Pop-up** | Making another kind of pop-up, which provides more complex motion, exploring symmetry, by making mirror-image pop-ups | 100 | arc, arc length, symmetric angle-fold, far side, link angle, radius, symmetric angle-fold, vertex | 1. Where does the Angle-fold hide?  2. Experiments with the Monster  3. Experiments with the Mouth | Observation, discussion, written work |
| 10 | **Peek-a-Boo and Handwaver** | Combining the two types of pop-ups to create surprising effects | 150 |  | 1. Experiments with the Peek-a-boo  2. Experiments with the Handwaver  3. The Twister | Observation, student projects |
| 11 | **Make a pop-up book** | Creating group pop-up projects | 100 |  |  | Observation |
| 12 | **The Pop-up Show** | Presenting pop-up projects to an audience | 100 |  |  | Student presentations |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Lesson** | **Title** | **Standards alignment** | | | | |
| **CCLS -- ELA** | **CCLS -- Math** | **NGSS – Scientific & Engineering Practices** | **NGSS – Crosscutting Concepts** | **NGSS – Disciplinary Core Ideas** |
| 1 | **Looking Closely at  Pop-ups** | **Writing**: Research to build and present knowledge **Speaking & Listening**: Comprehension and collaboration | MP7: Look for and make use of structure  4. OA5 & 5.OA3: Generate and analyze patterns and relationships | 1. Asking questions and defining problems  7. Engaging in argument from evidence 8. Obtaining and evaluating information | 1. Patterns 2. Cause and effect: mechanism and prediction 6. Structure and function |  |
| 2 | **Make a  Pop-up** | **Writing**: Text types and purposes; Research to build and present knowledge **Speaking & Listening**: Comprehension and collaboration **Language**: Vocabulary acquisition and use | MP7: Look for and make use of structure | 1. Asking questions and defining problems  3. Planning and carrying out investigations  6. Designing Solutions 8. Obtaining and evaluating information | 1. Patterns 2. Cause and effect: mechanism and prediction 6. Structure and function | ETS1: Engineering Design |
| 3 | **The Shape  of a  Pop-up** | **Writing**: Text types and purposes;  **Speaking & Listening**: Comprehension and collaboration **Language**: Vocabulary acquisition and use | MP2: Reason abstractly  MP7: Look for and make use of structure  4. OA5 & 5.OA3: Generate and analyze patterns and relationships  5G:3 &4: Classify 2D shapes into categories | 1. Asking questions and defining problems  3. Planning and carrying out investigations  4. Analyzing and interpreting data  8. Obtaining and evaluating information | 1. Patterns 2. Cause and effect: mechanism and prediction 6. Structure and function 7. Stability and change | PS2: Motion and stability: forces and interactions |
| 4 | **Finding Patterns in the Folds** | **Writing**: Text types and purposes; Research to build and present knowledge **Speaking & Listening**: Comprehension and collaboration **Language**: Vocabulary acquisition and use | MP2: Reason abstractly  MP4: Model with mathematics MP6: Attend to precision  MP7: Look for and make use of structure  4. OA5 & 5.OA3: Generate and analyze patterns and relationships | 1. Asking questions and defining problems  3. Planning and carrying out investigations  4. Analyzing and interpreting data 7. Engaging in argument from evidence  8. Obtaining and evaluating information | 1. Patterns 2. Cause and effect: mechanism and prediction 6. Structure and function |  |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Lesson** | **Title** | **Standards alignment** | | | | | | | | |
| **CCLS -- ELA** | **CCLS -- Math** | | | **NGSS – Scientific & Engineering Practices** | | **NGSS – Crosscutting Concepts** | | **NGSS – Disciplinary Core Ideas** |
| 5 | **The Pop-up Computer** | **Speaking & Listening**: Presentation of knowledge and ideas **Language**: Vocabulary acquisition and use | MP 2: Reason abstractly  MP3: Construct viable arguments MP4: Model with mathematics MP6: Attend to precision MP7: Look for and make use of structure  4. MD4: Represent and interpret data | | | 2. Developing and using models; 3. Planning and carrying out investigations  4. Analyzing and interpreting data 7. Engaging in argument from evidence  8. Obtaining and evaluating information | | 1. Patterns 6. Structure and function | |  |
| 6 | **Where does it Hide?** | **Writing**: Text types and purposes; Research to build and present knowledge **Speaking & Listening**: Comprehension and collaboration **Language**: Vocabulary acquisition and use | MP 2: Reason abstractly  MP3: Construct viable arguments MP4: Model with mathematics MP6: Attend to precision MP7: Look for and make use of structure  4. & 5.OA: Generate and analyze patterns and relationships  4. MD Represent and interpret data | | | 1. Asking questions and defining problems  2. Developing and using models; 3. Planning and carrying out investigations  6. Constructing explanations 7. Engaging in argument from evidence  8. Obtaining and evaluating information | | 1. Patterns 2. Cause and effect: mechanism and prediction  4. Systems and system models 6. Structure and function | | PS2: Motion and Stability |
| 7 | **Stair-step  Pop-ups** | **Writing**: Research to build and present knowledge **Speaking & Listening**: Comprehension and collaboration **Language**: Vocabulary acquisition and use | MP3: Construct viable arguments MP4: Model with mathematics MP6: Attend to precision MP7: Look for and make use of structure   5G: Classify 2D shapes | | | 2. Developing and using models  6. Designing Solutions 7. Engaging in argument from evidence 8. Obtaining and evaluating information | | 1. Patterns 2. Cause and effect: mechanism and prediction 4. Systems and system models 6. Structure and function 7. Stability and change | | PS2: Motion and stability: forces and interactions |
| 8 | **How they Move** | **Writing**: Text types and purposes; Research to build and present knowledge **Speaking & Listening**: Comprehension and collaboration **Language**: Vocabulary acquisition and use | MP3: Construct viable arguments MP4: Model with mathematics MP6: Attend to precision MP7: Look for and make use of structure  4. & 5.OA: Generate and analyze patterns and relationships 4. & 5. G: Draw and identify lines and angles | | | 1. Asking questions and defining problems  3. Planning and carrying out investigations 6. Constructing Explanations 7. Engaging in argument from evidence  8. Obtaining and evaluating info | | 1. Patterns 2. Cause and effect: mechanism and prediction 6. Structure and function | | PS2: Motion and stability: forces and interactions |
| **Lesson** | **Title** | **Standards alignment** | | | | | | | | |
| **CCLS -- ELA** | | **CCLS -- Math** | **NGSS – Scientific & Engineering Practices** | | **NGSS – Crosscutting Concepts** | | **NGSS – Disciplinary Core Ideas** | |
| 9 | **Angle-Fold Pop-ups** | **Writing**: Text types and purposes;  **Speaking & Listening**: Comprehension and collaboration **Language**: Vocabulary acquisition and use | | MP4: Model with mathematics MP6: Attend to precision MP7: Look for and make use of structure  4. & 5.OA: Generate and analyze patterns and relationships 4.MD5, 6 & 7: Understand concepts of angle and measure angles 4. & 5.G1, 2 & 3: Draw and identify lines and angles | 1. Asking questions and defining problems  3. Planning and carrying out investigations 4. Analyzing and interpreting data 6. Designing Solutions  8. Obtaining and evaluating information | | 1. Patterns 2. Cause and effect: mechanism and prediction 6. Structure and function | | ETS1: Engineering Design | |
| 10 | **Peek-a-Boo, & Hand-waver** | **Writing**: Text types and purposes; Research to build and present knowledge **Speaking & Listening**: Comprehension and collaboration | | MP4: Model with mathematics MP6: Attend to precision MP7: Look for and make use of structure  4. & 5.OA: Generate and analyze patterns and relationships 4.MD5, 6 & 7: Understand concepts of angle and measure angles 4. & 5.G1, 2 & 3: Draw and identify lines and angles | 1. Asking questions and defining problems  3. Planning and carrying out investigations  6. Constructing explanations and designing Solutions  8. Obtaining and evaluating information | | 1. Patterns 2. Cause and effect: mechanism and prediction  4. Systems and system models 6. Structure and function | | ETS1: Engineering Design | |
| 11 | **Make a group  pop-up book** | **Writing**: Text types and purposes; Research to build and present knowledge **Speaking & Listening**: Comprehension and collaboration | |  | 1. Asking questions and defining problems  3. Planning and carrying out investigations  6. Designing Solutions  8. Obtaining and evaluating information | | 1. Patterns 2. Cause and effect: mechanism and prediction  4. Systems and system models 6. Structure and function | | ETS1: Engineering Design | |
| 12 | **The Pop-up Show** | **Speaking & Listening**: Presentation of knowledge and ideas **Language**: Vocabulary acquisition and use | |  | 3. Planning and carrying out investigations 6. Designing Solutions | | 2. Cause and effect: mechanism and prediction 6. Structure and function | | ETS1: Engineering Design | |

Teaching Strategies

**Learning:** People learn by doing, and then reflecting on what they have done. In engineering, the goal is to design and create something new, and new designs rarely work well the first time. The effort to troubleshoot and fix something that doesn’t work provides rich motivation for learning. This curriculum unit provides numerous opportunities for students to explore for themselves, make things based on what they have learned, and reflect on their work in both oral and written form. Just as there is no one way to design something new, there is no one way to teach this unit. Be creative and flexible, and your students will be too!

**Vocabulary:** Words are not very meaningful unless they are connected with concepts. For this reason, we do not believe in presenting vocabulary words at the beginning of a lesson. Provide students with experiences that allow them to develop the concepts for themselves, and encourage them to use *their own words* to describe these concepts. *Then* provide the words that professional scientists and engineers use to describe these same concepts. These are the words provided in the Vocabulary column of the curriculum maps and the Word Bank section of each lesson. The Glossary at the end of this unit provides a working definition for each word.

**Writing and Drawing:**

Writing and drawing are essential parts of engineering design. The person who created something new is the only person who can describe what they did, and may be strongly motivated to convey these original ideas to others. This curriculum unit provides numerous opportunities for students to make sense of what they have done through text and graphics. They are encouraged to describe what they plan to make, the issues that prevented it from working, how someone else could make it, how it works and what was learned from it.

As much as possible, students need to express themselves in their own words (see Vocabulary, above), with no more prompts than necessary to get them started. The boxes labeled Science Notebook and the worksheets in the lessons provide starting points. These can be used in any combination, and students should also have opportunities to do more open-ended writing, for example to reflect on how they feel about their work.

Science Notebook entries are boxed.

* Writing prompts have lightning bullets.

Writing in notebooks and worksheets is primarily for the students themselves – to help them consolidate and remember what they have learned and communicate it to others – but it also serves as an assessment tool. It should not be marked closely for grammar and spelling. However, it is appropriate to ask students to read what they have written to the class, and to challenge them to clarify ideas that are unclear or incomplete. Much of the description will require drawings or diagrams as well as text, and it is important to help students learn to coordinate these two modes of communication.

**Discussion**:

Speaking and listening are essential forms of literacy and are central to learning science and engineering. The purpose of a discussion, like that of writing and drawing, is to create meaning. A discussion is not a question-and-answer session led by the teacher, nor a sharing session in which students simply report on what they did. Making meaning requires that students listen and respond to one another’s ideas. In a discussion of engineering design, students may want to bring up issues that they have encountered. Other students may respond by identifying similar issues, and/or by suggesting solutions that they have come up with. As the teacher, your role is to facilitate this give-and-take, by posing questions for discussion and then maintaining focus within the group. Sample focusing questions are identified like this within each lesson:

* *Lightning bullets and italics indicate prompts for discussion*

**Reference:** Worth, K., Winokur, J. Crissman, S., Heller-Winokur, M. (2009) The Essentials of Science and Literacy: A Guide for Teachers. Portsmouth, NH: Heinemann.

Structure of the Lesson Plans

The following categories appear in each lesson (\*), or most lessons (\*\*):

**\*Essential Question**

**\*Task**

**\*Standards**

**\*Outcomes**

**\*Assessment**

**\*\*Advance Preparation**

**\*Materials**

**\*Procedure**

**\*\*Word bank**

**\*\*Homework**

**\*\*Extensions**

**\*\*Worksheets**

Overview of Basic concepts

A **mechanism** is a device that has moving parts. A linkage is a type of mechanism that includes rigid arms called **links** or **bars**, joined by **pivots** (also called **joints** or **hinges**) that allow the links to rotate. The basic pop-up mechanism is called a **four-bar linkage,** because it consists of four links, joined by four pivots. Four-bar linkages are very common. You can find them inside the top of an umbrella, holding the trays of a sewing box or a tool box, on the sides of a shopping cart, supporting swing-arm desk lamps or extendable bathroom mirrors and in construction equipment. Figure 1 shows some other examples:



Figure 1: Four-bar linkages: fireplace tongs (left); scissor jack (right)

In a pop-up, each link is made from a stiff piece of paper. The hinges are just folds in the paper, or places where one piece is taped to another, so one piece can swing freely relative to the other. A pivot that works like a fold in paper is called a **hinge**. Hinges and links are the fundamental building blocks of pop-ups. To operate a pop-up, we provide a **force** to one of the links, usually by opening a page of a book or card. This force is transmitted by the mechanism to other links, causing them to move too. We see this motion in the piece that pops up, often unexpectedly, creating exciting visual effects.

Teacher Notes on the Lessons

Lesson 1: Looking Closely at Pop-ups

**Themes for Pop-ups**

Here are some ideas that teachers have used for classroom or group themes, sorted by subject area:

|  |  |
| --- | --- |
| English Language Arts | Author Study  Favorite character or scene from a book  Fantastic character  Autobiography (including self-portrait pop-up)  Fable or fairy tale  Tall tale |
| Social Studies | Neighborhood walk  Representing a culture or society  A historical moment or event |
| Science | Colors  Human body  A living thing  A biome  The Solar system  Weather events  Food chains |
| Math | Geometric shapes  Proportion and scale |

**Structure and function**

1. What do you have to do to make something pop up? This question may be a little confusing. Students may think you’re asking how to make a pop-up book. Explain that you’re talking about a pop-up book that someone has already made. What do you need to do to it to make something pop up? With this clarification, most students will recognize that **the motion of the pop-up is caused by opening and closing the book**.

2. What parts does a pop-up have? Students should notice that a pop-up needs to be made from at least two pieces of paper. In the simplest pop-up, one piece is inside the book, and the other piece forms the book itself.

Sometimes, the inside part and the outside part are made from the same piece of paper, but this leaves a hole in the back, as shown below:



The hole must be filled by a second piece of paper, so there still need to be two pieces

3. How do the parts need to be attached or arranged to cause the pop-up action?

This is the hardest of the three questions. If necessary, help students focus on whether and how the two pieces need to be connected in order for the pop-up to work. Call their attention to:

* Where things are folded;
* The directions of the folds (inwards or outwards);
* How the pieces are attached: in how many places, where and why.

They might observe that:

* The book has to fold inwards (away from you) as you close it.
* The pop-up has to fold outwards (towards you) as the book closes.
* In order to work, the pop-up needs to be attached to the book (page positions) on both sides of its center fold (gutter).

The reason for needing two attachments is that the book has to transmit a force to one side of the pop-up piece, which must be resisted by the book on the other side. Suppose we keep the right side of the book fixed, and move the left side only. Try making a pop-up by attaching the pop-up piece to the left side only. Then disconnect it and attach it to the right side only.

What goes wrong in each case? If the pop-up is attached only to the left side, the pop-up piece won’t be anchored, so it will simply follow the left side as in Figure a), below. On the other hand, suppose it’s attached only on the right side, as in Figure b). When the book opens, the pop-up won’t follow it, because the force of the moving page won’t be transmitted to it. To achieve pop-up action, the pop-up piece has to be attached to both sides, because it needs to be both anchored to the fixed side, and pushed or pulled by the movable side, as shown in Figure c). The attachment of the pop-up to the book can be made using tape, glue, or tabs-in-slots.



Lesson 2**:** Make a Pop-up

It is helpful to organize the Issues into two separate lists: (a) those that are relatively easy to solve, and (b) those that are not. On the first list, leave room for another column on the right, to show what they did (or would do) to solve the problem. The table 1 has some examples.

|  |  |  |
| --- | --- | --- |
| **Issue** | **Causes** | **Fixes** |
| Things come apart | Tape comes off | * Use wider or longer tape. * Place tape so it sticks to both pop-up and book. * Press tape harder. |
| Pop-up piece is too small to stick. | Use a bigger piece. |
| Paper bends too much. | Paper is not stiff enough | Use cardstock |
| Motion is not smooth | Things get in the way of each other. | Start simple – use fewer pieces. |
| Won’t pop up. | * Pop-up is attached on one side only * Tape crosses gutter * Fold is pointing inwards | * Attach pop-up on both sides of the gutter. * Don’t tape across gutter. * Make sure fold is pointing outwards. |

Table 1: Pop-up issues that are not hard to fix

While many of the issues in making pop-ups will give way to troubleshooting, there are also some that will require more knowledge to figure out. These will be the basis of the next set of activities, which involve more systematic investigations of the parallel-fold, or which require the other type of pop-up, the angle-fold. Table 2 lists examples of this second category of issues, along with the lessons that provide clues about how to solve each one.

|  |  |
| --- | --- |
| Issue | Lesson(s) that address this issue |
| Book won’t close without making a new fold | 3, 4 & 5 |
| Pop-up sticks out when book is closed | 5 & 6 |
| Pop-up should lie flat (instead of standing up) when book is open | 4, 5 & 6 |
| Pop-up should stand up (instead of lying flat) when book is open | 4, 5 & 6 |
| Pop-up should move up-and-down instead of back-and-forth | 8, 9 & 10 |
| It should be more surprising | 10 |
| Stuff should spring out beyond the book when it opens | 10 |
| Would like to have more than one pop-up in the same folder | 7, 9, 10 |
| Would like to make a pop-up that makes another one move | 7, 10 |

Table 2: Pop-up issues that will be addressed later in the unit

Lesson 3: The Shape of a Pop-up

Sometimes the pop-up piece won’t stay attached to the book. The problem is usually in the way the tape was applied. Figure 1 shows some Do’s and Don’t’s.

The tape should run along the edge of the strip, with as much surface area as possible attaching it to both the book and the strip, as in Figure 1 a). If the tape is applied cross-wise, as in Figure 4 b), it may not stick enough to hold the pieces together. If the tape is too long, as in Figure 4 c), it will prevent to pop-up from rotating, resulting either in additional folds, or the tape coming loose. Also, the edge of the strip should be taped parallel to the ruler lines, or the pop-up may be forced to twist or come loose. Figure 4 d) shows a pop-up that does not meet this guideline.



Figure 1: How to apply the tape: a) good technique; b) tape applied cross-wise;   
c) too much tape; d) pop-up piece is oblique to ruler lines

Closing the book always causes a fold to form in the pop-up piece. Even if the pop-up is folded beforehand, the book usually ignores this fold and makes a new one. How does it decide where to make the fold? Actually, it solves a math problem, which is developed in Lesson 5, and explained in Lesson 6. The next lesson prepares the way to see this problem, through two qualitative investigations.

Lesson 4: Finding Patterns in the Folds

1. Near and Far: In the first experiment, there are two **controlled variable**s:

* the distance between page positions is always 10 cm.
* the strip length is always 12 cm.

One page position is the **independent** (or **input**) **variable** (the other page position is always 10 cm. away). The location of the resulting fold line is the **dependent** (or **output) variable**.

Each pop-up has a **near side** (closer to the gutter) and a **far side** (further from the gutter), except for #3, which is symmetric. The experiment shows that closing the book always forces **the fold line to be on the far side**, unless the pop-up is symmetric, in which case the fold line will be centered.

Figure 1 shows the outcome of the experiment, with the near side and far identified for pop-ups #1 and #5.



Figure 1: It always makes the fold on the far side

Here is an explanation of this pattern:

* Pop-up #3 is symmetric, so both page positions are equal. The fold divides the strip up the same way, so the fold happens in the middle.
* In #1, we’ve made the left page position much further than the right one. To compensate, the book makes the right side of the pop-up much shorter than the left side, which forces the fold over to the right side.
* #5 is exactly the opposite of #1. There, the fold is pushed over to the left side, to compensate for the much longer page position on that side.

2. High and low: In this experiment, the two page positions are equal. There are several **controlled** **variables**:

1. The two page positions are equal.
2. The strip is 12 cm. long.

As we saw from the pop-up #3 of Near & Far the pop-up piece will fold in the middle when the page positions are equal. That is true of all five pop-ups in High & Low.

The **input variable,** or **independent variable,** is either page position. Students will probably notice that the pop-up rises higher above the page if both sides are taped closer to the gutter. You might ask them, so what **variable** changes as a result of moving the places where both sides are taped (page positions) closer to the gutter? The most obvious **output variable,** or **dependent variable,** is how high the fold line rises above the open book. Figure 2 shows pop-ups # 2- #5 after the book opens.



Figure 2: What changes when strips are taped near or far from the center?

Why does moving the page positions inward force the pop-up to rise higher? Remember, all five strips are the same length. If one strip is taped closer to the gutter than another, it will not be able to lie as flat because it won’t have as much space near the page. Therefore it will have to stand higher.

Lesson 5: The Pop-up Computer

As students are reporting their data, scan each row quietly to verify:

A + B = C + D and B + C = 12

If both are true, it is probably good data. If a row of data meets both tests, put a star next to it, and say, “That’s good data.” Enter data on the chart until you have about 5 or 6 rows of good data. Then ask students to look for patterns in the data. What were you looking for when you decided that data was “good?” If students have difficulty seeing the pattern, they can try to find it using Part B of the Worksheet. Once they have found the pattern, they should describe it in Part C.

There are several common reasons for bad data:

1. Failure to cut the strips properly. Each strip should be approximately 12 cm. long. There are marks at 1 through 11, but not at 0 and 12. Students may have cut them elsewhere besides 0, but assumed the numbers were valid anyway.
2. Students count lines rather than boxes on the rulers. This is a classic measurement problem. Review the use of a ruler.
3. Failure to read B and C independently. Students may always show the same number for B and C. This is the subject of the Mini-Lesson, “Measuring Link Lengths” in Procedure Step 3.
4. The actual hinge is not located at the measured page position line. Recall that the four significant lines in a pop-up are the hinges. If too little tape is used, it may lift off the book, and a new fold may form in the tape itself (Figure 2 (a)). If too much tape is used, the pop-up may form a new hinge line, because the tape won’t allow a fold at the measured line (Figure 2(b)).
5. The page position line is not parallel to the gutter. In this case, *no* measurement of the link lengths can be valid, because these will vary along the width of the strip. See Figure 2c). Encourage students to check whether the page position lines are parallel to the gutter, and ruler lines in the book.
6. Round-off error. If none of the above issues applies, but the results are still off by 1 or 2 cm., the problem may simply be due to round-off. Students may count only whole numbers of cm. Encourage them to estimate and use half-centimeters.

Extensions:

## 1. How can you make it lie flat or stand up?

When a pop-up lies flat, it looks like the diagram in Figure 1.



Figure 1: Link lengths labeled on a pop-up that lies flat when book is open

From the diagram, you can see that A + D = C + B. Also, for the book to close, A + B = C + D. The only way both of these can be true is for both

A = C and B = D.

2. What happens if you break the rule?

If you make a pop-up that doesn’t have A + B = C + D, everything will be fine until you try to close it. Then it will typically either make a new fold, forcing A + B = C + D with the new links and fold line; or the tape may come off, because it wasn’t tight enough to make the new fold form.



Figure 2: Taping problems that can lead to invalid measurements

Lesson 6**:** Where Does it Hide?

1.When does it make A + B = C + D?Until the book actually closes, it doesn’t matter what A, B, C & D are. The book forces the fold between B and C when it closes. This is the fold that enforces the rule A + B = C + D, which is the condition for the book to close.

2. Where does it hide**?** Below is an Answer Guide to the Worksheet, which shows the approximate location of the pop-up in each of the three cases.



Figure 1: Answer guide to Where Does it hide?

## 3. The see-thru book The pop-ups inside the See-thru book don’t need to be exact. The predictions are intended to be approximate. The pop-up in row a) should be approximately symmetric; row b) should have a larger page position on the right than on the left (A > D), and the opposite should be true in row c) (A < D). The pop-ups should be assembled without first folding the strip. If a strip doesn’t pop out, lift it slightly as you close it to make sure the fold is pointing towards you.

4. Explaining where it hides.The explanation depends on which hinge lines are moving and which are not as the book closes. To keep track of these, it may be helpful to label them on the See-thru Book, as in Lesson 5. The table below shows what happens to each hinge and each link as the book closes:

|  |  |  |  |
| --- | --- | --- | --- |
| **Link** | **Motion of its hinges** | | **Motion of link** |
| A | gutter does not move | right page position does not move | A (right page) does not move |
| B | right page position does not move | fold moves to the right | B flips over right page position and follows fold line to edge of right page |
| C | fold moves to the right | left page position moves to the right | C follows left page and moves above B |
| D | left page position moves to the right | gutter does not move | D (left page) flips around gutter and moves above A (right page) |

5. Why does A + B = C + D?Figure 2shows theSee-thru book from a top and bottom view**,** with all four link lengths labeled. From the figure, it should be clear that A & B will lie side-by-side on the bottom and C & D will lie side-by-side on the top. To see the bottom view, look at the folder from underneath. If you flip it over, you’ll see a mirror image. Both distances, (A + B) and (C + D), extend from the gutter to the fold line, and both are straight lines, so the two distances must be the same.



Figure 2: Inside the see-thru book

Extensions:

1. Pop-ups that stay inside the book Figure 3 shows the three cases inside the See-thru Book:



Figure 3: Top view of the see-thru book: a) pop-up stays inside; b) just touches or c) sticks out

From the diagram, the conditions for the three cases are:

1. The pop-up will stay inside if C + D (or A + B) is less than W.
2. The pop-up will just touch the right edge if C + D (or A + B) = W.
3. The pop-up will stick out if C + D (or A + B) is greater than W.

2. Geometry through pop-ups

From Extension 1 to Lesson 5, a pop-up that lies flat when fully open has A = C and B = D. These are the sides of a rhombus. Therefore, a pop-up that lies flat when the book is open will look like a rhombus See Figure 4. 

Figure 4: Edge-view of a pop-up when A = C and B = D

A rectangle is just a special case of a rhombus when all four angles are 90○, as in Figure 4 b). If a pop-up that lies flat is also symmetrical, then A = C and B = D, as before, but also A = D, so all four sides are equal. When the angles are 90○, as in Figure 4 b), the rectangle becomes a square, which is just a rectangle with 4 equal sides.

Lesson 7: Stair-step Pop-ups

Systems**.** With a little help, students should recognize that the inputs to a video game are through the controls. The actions might include pushing buttons, moving a joystick or turning a trackball. The outputs are the sound effects and motions of figures on a screen. Wii systems often have a wider range of inputs, such as dancing, playing a musical instrument or using fitness equipment. Students are likely to come up with other examples, such as the computer, whose input includes the motion of the mouse. The input to a pop-up mechanism is the opening and closing of the book. The output is the motion of the pop-up piece itself. These relationships are expressed in Figure 1.



Figure 1: System diagram of a simple pop-up

Edge view**.** Figure 2 below shows the half- and quarter-open positions. Viewed from the edge, the shapes inside the two pop-ups are rhombuses, which become rectangles when the book is half open.



Figure 2: Edge view of stair-step pop-up in a) half- and b) quarter-open positions

System diagrams. The key to making the system diagram for the Stair-step Pop-up is to recognize the input of each of the two pop-ups. The first pop-up is controlled by the book, but the second pop-up is attached to the book only on one side. Its left page position is on the first pop-up. Therefore the input to the second pop-up is really the first pop-up. This leads to a system diagram like in Figure 3.



Figure 3 System diagram of a Stair-step Pop-up

Extensions

Add more pop-ups: If two pop-ups are assembled by attaching each one directly to the book on both sides, as in Figure 4, there will again be two subsystems, but they will not operate like a stair-step pop-up.



Figure 4: Two pop-ups operated directly by the book

The reason is that they both receive their inputs directly from the book, not one from another, as in the stair-step. Therefore, the system diagram is different.



Figure 5: System diagram for pop-ups in Figure 4

The arrangement in Figures 4 and 5 is called a **parallel connection**, because both subsystems receive the same input. The stair-step pop-up in Figures 2 and 3, on the other hand, is a **series connection**, because the output of one is the input to the next. These terms have the same meaning here as in electric circuits. Disconnecting one bulb in series with others, as in some Christmas tree lights, will make the others go out. However, disconnecting a bulb that is in parallel with others, as in the lights above a bathroom cabinet, will have no effect, because all are attached directly to the same power source. Similarly, disconnecting Pop-up #1 in the series connection of Figure 3 will disable Pop-up #2, but have no effect on Pop-up #2 in the parallel connection of Figures 4 and 5.

Lesson 8: How they Move

Types of pop-up motion:

Students may have difficulty seeing all three dimensions of motion. The easiest one to leave out is 3D (towards and away from the book), because all pop-ups display that kind of motion – that’s why they seem to “pop up!” Type A pop-ups have horizontal and 3D motion, while Type B pop-ups have all three: horizontal (back and forth), vertical (up and down) and 3D (in and out). In a Type B pop-up, the vertical motion is the most obvious, because most of it happens just as the book opens completely, making it the last thing you see. See Figure 1.



Figure 1: Two basic types of pop-up

Lines in space and hinge lines in pop-ups

These are easy to illustrate using a medium-size cardboard box, like the kind that copy paper is shipped in. See Figure 2. The hinge lines in the two types of pop-ups are shown in Figure 3.

Figure 2: Lines in Space



Figure 3: Hinge lines in both types of pop-ups

Extension: A pop-up whose hinge lines are skew.

Why can’t the hinge lines in a pop-up be skew to one another? You *can* make a pop-up in which the hinges are skew. In 3D, there’s no problem. But when you close the book, there’s one dimension less. A closed book is 2D, like a flat surface, and there are no skew lines in 2D. So, if you make a pop-up that has gutter, page position and fold lines that are skew, *you won’t be able to close it, unless either:*

* New folds are forced to form when you close the book; or
* There are twists in the pop-up piece when the book is open.

In most pop-ups, the paper is not twisted, and the pop-up pieces are flat planes between folds. That’s why there are only two basic kinds of pop-up: the four hinge lines either have to run parallel or intersect in a single point, the vertex.

Lesson 9: Make an Angle-fold Pop-up

The Basic Angle Fold: Here is a troubleshooting chart showing issues, possible causes and fixes.

|  |  |  |
| --- | --- | --- |
| **Issue** | **Possible cause** | **Fix** |
| Everything comes apart | Not enough tape | Use more tape |
| Pop-up gets hung up | See next three rows |
| Book won’t close easily | Book or triangle does not have a sharp fold | Score, fold and burnish along fold line |
| Triangle vertex not touching gutter | Move triangle so its tip touches gutter |
| Tape gets in way of triangle folding | Use less tape |
| Inside won’t pop-up | Fold points towards book | Reverse fold by lifting with finger |

Table 1: What can go wrong with an angle-fold

Where can you get the most motion?

The maximum motion occurs at the flat edge of the triangle, furthest from the page positions and vertex (see Figure 1. The reason is that the page positions and vertex are taped down, and therefore can’t move. The vertex acts like the axis for the rotation. Like on a merry-go round or see-saw, further you are from the axis, the further you travel.



Figure 1: Where to put the flag in order to increase the amount of motion

To amplify the motion even further, attach a flag along the fold line, extending outwards away from the vertex. Why does this work? The flag is like a baseball bat and the vertex is like the shoulders of the batter. The bat and the flag both extend the swing because they are further from the place where the swing starts – the vertex. The distance from the vertex is called the **radius**, and the shape of the swing is an **arc**.

**2. Assemble it unfolded**: Just like a basic parallel-fold, an angle-fold pop-up has four links, separated by the four hinges. When you assemble an angle-fold without folding the triangle first, you select the two page positions by where you tape it down. However, we can no longer measure the lengths of these two links, because they have to intersect each other and the gutter at the **vertex**, where the distance between any of these lines is zero. Then the lines extend outwards, so their distance from each other increases. The quantity that does stay the same as two intersecting lines get further apart is the **angle** between them. The size of an angle measures how rapidly they spread apart as you move away from the vertex.



Figure 2: Assembling it unfolded: Before (left) and After (right)

When you close the book and then open it again, it makes a fold in the triangle – just like the parallel-fold does in the strip. It “decides” where to make the fold, based on the same formula, A + B = C + D, except that now A, B, C and D are angles, not lengths.

Reversing the Triangles

When you reverse the two angle-folds, the two horizontal edges will still produce the motion. As the book opens, the bottom one will go up and the top one will go down, so they will come together, like upper and lower jaws. You have probably already seen this construction used in commercial pop-up books to make a **mouth**!



Figure 3: How the Monster and Mouth move as the book is opened

Extensions

Where will the angle-fold hide?

You probably have to try this to believe it. Figure 4 shows what happens to the same angle fold that lay across the gutter when the book was open, Figure 6 in the Lesson. When the book is closed, it flips over to the right!



Figure 4: Where the angle-fold hides when the book is closed

This exercise is similar to the Worksheet in Lesson 6, and the reason for the answer is similar too. Figure 5 shows how the fold line moves across the right page position, from a vertical to a horizontal position, as the book closes. To see it for yourself, construct an angle-fold inside a See-thru Book.



Figure 5: How the fold moves across the right page position as the book closes

The flag follows the fold line, so it too gets reflected across the right page position and winds up in a horizontal position. When the book opens, the reverse happens. The flag always travels towards the side of the triangle opposite the vertex.

Figures 6 & 7 show what happens in the Monster and the Mouth.



Figure 6: Where the Monster hides when the book is closed



Figure 7: Where the Mouth hides when the book is closed

2. Experiments with the Monster:

* + 1. The longer the flag, the more it will appear to move.
    2. The C-shaped flags are very surprising – you have to try it to believe it! The bottom of the upper “C” travels down as the book opens, while the top of the lower “C” goes up. To see why, make the same pop-up inside the clear folder, and note carefully how the top and bottom of each “C” moves as the book opens and closes.
    3. Figure 8 shows two identical angle-folds in the same-width book, but with different-length flags. Of the two, only the flag on the right sticks out of the book when it is closed. Why? This type of angle-fold will stick out of the book or not depending on how the total radius (distance from the vertex) of the end of the flag compares with the width of the book.



Figure 8: How the flag length affects whether or not it will stick out

4. Experiments with the Mouth:

b. To make the tongue move with one of the jaws, attach it on the inside of the jaw. To make a tongue that doesn’t move with a jaw, attach it to the book.

e. Figure 9 below shows how to make asymmetric angle folds.



Figure 9: How to make an asymmetric angle-fold

Using asymmetric angle-folds for the top and bottom jaws will make the mouth appear crooked when it is open.

Lesson 10: Peek-a-Boo and Hand-waver

Peek-a-boo

Figure 1 shows how the Peek-a-Boo should work.



Figure 1: Peek-a-boo in action

Troubleshooting Tips

* Attach the triangle (angle-fold) first, so that it will be inside the rectangle (parallel-fold).
* If the angle-fold doesn’t work properly, follow the troubleshooting hints in Lesson 9.
* Make sure the fold in the rectangle (parallel fold) is pointing outwards (mountain fold). If necessary, pull it gently as the book begins to close to make it extend towards you.
* If the flag gets crushed or bent, you can either trim it so it fits properly, or else make the parallel-fold piece larger.

Hand-waver

Figure 2 shows how the Hand-waver should work.



Figure 2: Hand-Waver in action

Troubleshooting Tips:

* Make the three folds in the proper sequence: mountain fold (towards you), valley fold (away from you), mountain fold (towards you).
* Burnish the three folds to make them sharp. This can be done in one step, after all three folds have been scored and made.
* If the flag gets crushed or bent, trim it so it fits properly.

Comparing the Hand-waver and the Peek-a-Boo

The Peek-a-boo uses an angle-fold and a parallel-fold in a **parallel connection** (both controlled directly by the book). The upward motion of the flag is along the gutter, just like in the basic angle-fold. The flag hides inside the parallel-fold until the last moment. See Figure 3 a).

The Hand-waver also uses a parallel-fold and an angle-fold, but the book is attached directly to the parallel-fold only. The angle-fold is controlled by the parallel-fold, in a **series connection**. The upward motion of the flag is on the right page of the book. See Figure 3 b). Students may have difficulty in seeing the angle fold inside the Hand-waver. To understand how it works, they should try Extension Activity 2 a.



Figure 3: How the flag moves as you open the book

Figure 4 shows a system diagram for each one.



Figure 4: System diagrams for Peek-a-Boo and Hand-Waver

Extensions

1. Experiments with the Peek-a-boo

* 1. The location of the parallel-fold is determined by the usual rule, A + B = C + D. The parallel-fold should be made to move as far as possible, in order to be able to hide the largest possible flag. To make the parallel fold move the maximum amount, it should lie flat when the book is open, which requires A = C and B = D (see Lesson 5, Extension 1). The template uses A = C = 8 cm. and B = D = 5 cm. If the parallel fold is much smaller, the flag will not fit inside. If it is bigger, the parallel fold may stick outside the book when it closes.
  2. The flag has to fit inside the parallel-fold, when the book is closed. Therefore, the total length of the flag + angle-fold, measured starting at the vertex, cannot be longer than link lengths C + D ( = A + B) of the parallel-fold. Figure 4 shows how the angle-fold and flag fit inside the parallel-fold when the book is closed.

  
Figure 5: How the angle-fold (dashed triangle) & flag (grey rectangle) fit inside the parallel-fold when the book is closed; C and D are parallel-fold link lengths

If the flag is too long, to fit, there are two options: shorten the flag or lengthen the parallel fold. However, there are some limits on how much longer the parallel fold can be:

* + - If B + C > A + D, the parallel fold won’t open flat.
    - If A, B, C or D increases by more than 1 cm., the parallel fold will stick out of the book when it closed, because A + B = C + D will be greater than the book width W = 14 cm.
    - If B + C > A + D, the parallel fold won’t open flat. See Lesson 10. Also, if B or C increases by more than 1 cm., the parallel fold will stick out of the book when it closed, because A + B = C + D will be greater than the book width W = 14 cm.

2. Experiments with the Hand-waver

a. Figure 6 compares a two-piece design (left) with the original design (right). The advantage of the two-piece design over the basic design is that the angle fold is hidden, like in the Peek-a-boo. The disadvantage is that it requires another piece of paper, and more taping. Otherwise they are very similar.



Figure 6: Comparing the two Hand-waver designs

b. Cut a wider piece for the parallel-fold, so it can fit angle-folds both above and below the center line. The bottom angle-fold should be a mirror image of the top one.

3. The Twister

How to Make It



Figure 7: Basic steps in making a Twister

Troubleshooting Tips

* If the pieces come loose, check the taping. All three pieces should be firmly attached. Each triangle should be attached to the book on both sides of the gutter, and the parallelogram should be attached to one of the triangles at either end.
* Make sure the tape extends all the way along the tape lines indicated, but not any further, because too much tape could interfere with the folding of the pieces. See Figure 7 a).
* Sometimes the tape will lift off of the parallelogram bridge, leaving a gap between it and the triangle. Reposition the bridge so the fit is as close as possible. See Figure 7 b) & c).
* If the parallelogram bridge doesn’t fold in it, close the book again, and press hard. The fold in the bridge should be near the center.
* If the rotation isn’t smooth, make sure all three folds (the two angle folds and the fold in the bridge) point outward, away from the book. Also, check the positions of the two triangles, to make sure they don’t collide with each other.

How it works

A Twister begins with two angle-folds in a parallel connection, exactly the same idea as the Monster. Both of these angle-folds are in a series connection with a parallel fold, the rhombus bridge over the top, because the two angle-folds operate the bridge from opposite ends. Figure 8 shows a system diagram



Figure 6: System diagram for the Twister

Materials for Pop-ups

|  |  |  |  |
| --- | --- | --- | --- |
| **Item** | **Detail** | **Qty** | **Lessons** |
| Reclosable storage bag | 2 gallon | 30 | 2-12 |
| Blank Cardstock | AstroBrights -- Assorted colors – half sheet (4 ¼ x 5 ½)” | 100 | 2-4 |
| AstroBrights -- Assorted colors – full sheet (8 ½ x 11)” | 50 | 7-12 |
| Masking tape | ¾” x 60 yd. roll | 6 | 2-12 |
| Templates (printed on cardstock) | Parallel –fold | 180 | 3-7 |
| Angle-fold | 90 | 9 |
| Peek-a-boo | 30 | 10 |
| Hand-waver | 30 |
| Twister | 30 |
| See-thru book | Cut and taped from transparency film | 30 | 6, 9 |
| Sample pop-up books | Simple (e.g., Pop-up Fun or Sesame Street series) | 6 | 1, 2, 8 |
| Intermediate (e.g., Snappy series or book by J. Pienkowski) | 1 | 7 - 12 |
| Complex (e.g., book by R. Sabuda or M. Reinhart) | 1 | 8 - 12 |
| **Supplied by school** | | | |
| Scissors |  | 15 | 2-12 |
| Rulers |  | 15 | 3-12 |
| Ball-point pens | These need not be working! | 15 |
| Post-its ™ or paper & glue sticks | Pack of Post-its ™ | 2 | 8-12 |
| Sample pop-up books | Simple (e.g., Pop-up Fun or Sesame Street series) | 10 | 1, 2, 8 |
| Intermediate (e.g., Snappy series or book by J. Pienkowski) | 7 | 7 - 12 |
| Complex (e.g., book by R. Sabuda or M. Reinhart) | 2 | 8 - 12 |
| **Craft Materials** | | | |
| Google eyes | assorted sizes | 100 | 7, 10-12 |
| Feathers | assorted sizes & colors | 100 |
| Foam stickers | assorted shapes & colors | 100 |
| Pipe cleaner | assorted colors, 12″ long | 60 |
| Cocktail umbrella | assorted colors, 4″ diam. | 30 |
| Pom-poms | assorted sizes and colors | 100 |

Notes about Materials

**Cardstock**: Ordinary paper or construction paper is not stiff enough for making pop-ups. Usecardstock**,** 65 lb. or higher, bright colors if possible. It is sold in packs of 250 by office supply stores. AstroBrights ™ by Wausau Paper Co. provides excellent colors and stiffness.

**See-thru book:** These are transparent folders made from clear transparency films, used to view the location of a pop-up when the book is closed. They are provided along with the templates as part of the curriculum unit. If additional see-thru books are needed, they can be made quickly using transparency films and clear tape. Directions for making a see-thru book are included on the next page.

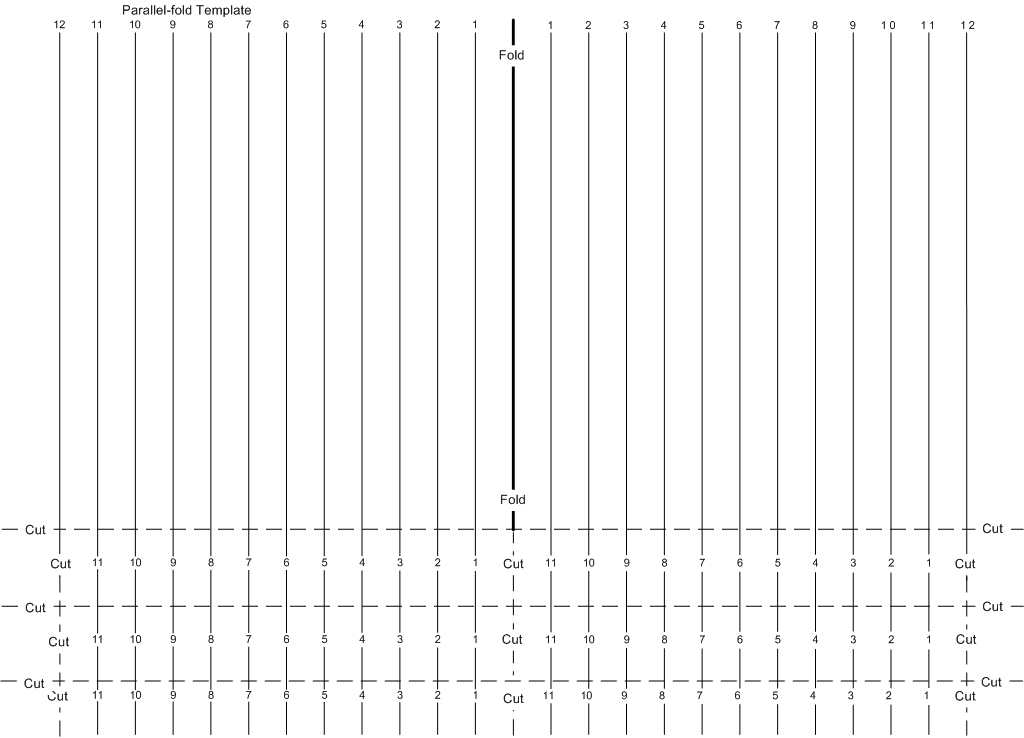
**Templates**: To get students started quickly with pop-up constructions, the unit provides five templates that have cutting, folding and taping lines printed directly on them. These are provided in the quantities shown in the table above. Additional templates can be printed from the masters on the pages following the Directions for making a See-Thru book.

**Commercially made pop-up books**: Students examine these in Lessons 1 & 8 to observe how pop-ups work, and get ideas for making their own. We focus on books in which the motion is powered simply by opening and closing the book. Books that use push- or pull- tabs, or wheels, to generate the motion are sometimes called “pop-ups” in bookstores, but we will not consider them in this unit. We classify pop-up books and their mechanisms as falling into three categories, which we’ll call **simple**, **intermediate** and **complex**:

* In a **simple** pop-up book, the construction method is easy to see and understand. The pop-up parts are directly attached to the book, and the motion is towards the center of the book, as in Figure 2 a) or b). These are widely available in Dollar stores and “big-box” discount stores and are usually very inexpensive.  
  Marvel Comics’ True Believers Retro Character Collection; and most pop-up greeting cards are also in this category.
* I**ntermediate** pop-ups are more difficult, but accessible once students have mastered the simple pop-ups. Intermediate constructions are compound mechanisms in which the output of one pop-up is the input to another. All of the student-made pop-ups displayed at <http://www.citytechnology.org/unit/pop-ups> are in this category. Intermediate pop-up mechanisms are found in books by Jan Pienkowski; Back Pack Books; Dr. Seuss pop-ups; and Snappy Books.
* **Complex** pop-ups are useful for getting students’ attention, but are mostly well beyond anything they could make themselves. In this category are books by David Pelham, David Carter; and especially Robert Sabuda and/or Matthew Reinhart.

Sample books are provided as part of the unit, and additional ones should be added. These are often obtainable from a library, teachers’ or children’s collections or discount stores.









# Lesson 1: Looking Closely at Pop-ups

## **Essential Question**

How does a pop-up work?

## **Task**

Examine a variety of pop-up books, to find out their basic parts and how these parts are connected.

## Standards:

CCLS – ELA **Writing**: Research to build and present knowledge  
**Speaking & Listening**: Comprehension and collaboration  
**Language**: Vocabulary acquisition and use

CCLS – Math  
**Math Practices**: MP7: Look for and make use of structure **Operations and Algebraic Thinking**: 4. OA5 & 5.OA3: Generate and analyze patterns and relationships

NGSS

**Scientific & Engineering Practices** 1. Asking questions and defining problems; 7. Engaging in argument from evidence; 8. Obtaining and evaluating information.  
**Crosscutting Concepts:** 1. Patterns; 2. Cause and effect: mechanism and prediction; 6. Structure and function

## Outcomes

* Students develop ideas for making their own group pop-up books
* The **force** needed to operate a pop-up is powered by **opening and closing the book.**
* A pop-up is a **mechanism** that needs two pieces of paper: one for the **book** and another for the **pop-up** piece. The pop-up piece has to be attached to the book in two places, one on either side of the main fold.

## Assessment

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Objective:** | **Below (1)** | **Approaching (2)** | **Proficient (3)** | **Advanced (4)** |
| A. Develop an idea for a group pop-up book | No idea | Basic framework, but few if any details | Clear idea, with ideas for each page | Clear idea, including sketches and suggestions for moving parts |
| B. Observe and recognize how a pop-up book works and is constructed | Little or no understanding | Recognizes that a book is powered by opening and closing but not aware of its construction | Can identify and name the basic parts of a pop-up and how they are attached | Can explain why the pop-up piece has to be attached to the book on either side of the gutter |

## Materials

* One complex pop-up book
* An assortment of simple pop-up books and/or cards, about 3 or 4 per group
* Science Notebooks

## Procedure

1. **What this unit is about** (Whole class – 10 min.) Demonstrate to the class the one **complex** pop-up book, opening each page several times slowly so that students can see the complex motions powered by the book. Ask:

* What is making this work?
* What materials would you need to make a book like this?
* What would you need to know?
* If you knew how to make a book like this, what could you represent?

Explain that in this unit, they will actually be creating their own pop-up books! But first, they will need to learn something about how pop-ups work and how to make them.

2. **Ideas for pop-up books and cards** (Small groups – 20 min.) Ask each group to come up with an idea for a pop-up book they would like to make as a group. If you would like these to be geared to a class theme, you can review it first.

3. **Sharing ideas** **for pop-ups** (Whole class – 20 min.) Each group reports briefly on their idea. Encourage members to contribute suggestions and ask questions.

**Suggested breakpoint between periods**

4. **Examining Pop-ups** (Small groups – 10 min.) Provide several **simple** pop-up books for each group to share. Ask:

* What do you notice about these pop-ups?

5. **Group reports on pop-ups** (Whole class – 10 min.) Convene the entire class, and ask each group to report one item at a time, going around the room as many times as necessary, until all the items have been reported. Each item should not duplicate those that have already been reported. Record all the items on the blackboard or chart paper for everyone to see.

6. **Looking closely at how they work** (Small groups – 20 min.) Many of the items reported in #2 are likely to be general observations or impressions, such as “they’re colorful,” “surprising” or “tell a story.” The purpose of this next activity is to analyze how the pop-ups work and how they are constructed. Explain that these are things we’ll need to know to make our own pop-ups. Develop the idea that a pop-up is a **mechanism**, because it has **moving** parts. In case these observations haven’t come up already, use these prompts:

* What do you have to do to make something pop up?
* How many parts does it need to have?
* Where are these parts folded?
* How are these parts attached?

The groups should look again at the pop-ups to try to answer th4se questions.

7. **Focus on Function and Structure** (Whole class – 10 min.) Ask the groups to report on their findings. Record their observations on the blackboard or chart paper. Some typical answers might be:

* To make it pop up, you have to open the book;
* It has moving parts;
* There’s an inside part and an outside part;
* It has folds;
* The pop-up part has to be attached to the book on both sides of the fold line.

Encourage students to draw on their observations when they answer the next three questions in their Science Notebooks or on the Worksheet, which can be attached to the Notebook. .

**Science Notebooks**

1. What parts did you notice in the pop-ups?
2. What do you have to do to the pop-up book to make something pop-up?
3. How do you think the parts should be attached to make it work?

## 

## Word Bank

Attachment, Book, Fold, Force, Mechanism, Page positions, Pages, Pop-up piece:

## Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date: \_\_\_\_\_\_\_\_\_\_\_\_\_

Lesson 1: **Examining a Pop-up**



1. What do you notice about the parts of a pop-up? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

2. Label the parts of a pop-up in the diagram above. Use your own names for them.

3. What do you notice about how these parts are attached to each other?

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

4. Why do you think they are attached this way? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

# Lesson 2: Make a Pop-up

## **Essential Question**

How can you construct something that can give you pop-up action?

## **Task**

Create a pop-up card using cardstock and tape

## Standards:

CCLS – ELA **Writing**: Text types and purposes; Research to build and present knowledge  
**Speaking & Listening**: Comprehension and collaboration  
**Language**: Vocabulary acquisition and use

CCLS – Math  
**Math Practices**: MP7: Look for and make use of structure

NGSS   
**Scientific & Engineering Practices** 1. Asking questions and defining problems; 3. Planning and carrying out investigations; 6. Designing Solutions; 8. Obtaining and evaluating information.  
**Crosscutting Concepts:** 1. Patterns; 2. Cause and effect: mechanism and prediction; 6. Structure and function  
**Disciplinary Core Ideas**: ETS1: Engineering Design

## Outcomes

* Troubleshooting is a systematic process in which you identify an issue, suggest what might be its cause, and then fix that one problem
* Use of standard names for the parts provides a common language for discussion of issues
* Pop-up mechanisms are easy and fun to make, if you proceed systematically and work together

## Assessment

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Objective:** | **Below (1)** | **Approaching (2)** | **Proficient (3)** | **Advanced (4)** |
| A. Develop troubleshooting techniques | Does not identify issue, cause or fix | Can identify issue, but not cause or fix | Identifies issue, cause and fix | Identifies multiple possible causes, and fix for each one |
| B. Use standard names to refer to pop-up parts | No use of standard names | Uses some standard names, but not others | Uses all standard names | Explains why use of standard names is helpful; evaluates alternative names |
| C. Create your own pop-up | Unable to make a pop-up | Creates a pop-up, but unable to replicate it | Creates a single type of pop-ups reliably | Experiments with alternative types of pop-ups |

## Advance Preparation

* Make a few pop-ups of your own to become familiar with the issues that arise; for guidance, view the video at <http://www.citytechnology.org/force-and-motion/make-a-pop-up>
* Create two identical charts showing the basic parts of a pop-up:



Figure 1: The parts of a pop-up

## Materials

* Books or cards with simple pop-ups, two or three per group
* Cardstock in a variety of colors, two or three sheets per student.
* Scissors, one or two per group
* Masking or cellophane tape, 1/2″ or 3/4″ wide, one roll per group
* Storage bags for saving pop-ups, one per student
* Charts from Lesson 1
* Science notebooks
* Markers or crayons, glue stick, Post-Its™

## Procedure

**1. Review of the Parts of a Pop-up** (Whole class – 15 min.) Present one of the two charts you have made showing the parts of a pop-up (see Figure 1, above). Develop the idea that it will be easier to talk about these parts if everybody agrees on their names. Ask:

* What should we call each part?
* What should we call the places where they are attached?
* What should we call the two fold lines?

Try to develop a consensus among the class about a common language for each of these items. If there are serious disagreements, the class can vote to resolve each one. Once there is an agreed-upon name, write it in on the chart.

Then tell the class that the people who create complex pop-ups, like the one you demonstrated in Lesson 1, are called **paper engineers**. They have the same problem as the class: they need a standard language too, for describing the parts of a pop-up. Introduce the standard language of paper engineering, by adding it to the second blank chart showing the parts of a pop-up:



Figure 2: Standard language of paper engineering

The class can then decide whether to use the language they have agreed upon, or the language used by paper engineers. Whichever they decide, keep the chart posted showing those names.

**2. Pop-up Principles** (Whole class – 15 min.)**:** Use the charts from Lesson 1 to review what was learned by examining pop-ups:

* A pop-up needs to have 2 pieces of paper, one for the book and the other for the piece that pops up;
* The pop-up piece has to be attached to the book in two places;
* There is a fold in each piece in between the places where they are attached. The pop-up fold points towards you while the fold in the book points away from you.

Using the agreed-on language, post these ideas prominently on a chart titled: Pop-up Principles. For example, in the language of paper engineers (Figure 2), these principles would read:

* A pop-up needs to have two pieces: a book and a pop-up piece.
* The two page positions have to be on either side of the gutter.
* The gutter points away from you and the fold points towards you.

3. **Making pop-ups** (Individual – 20 min.): Provide materials and simple pop-up books to each group. Each student’s task is to make a working pop-up. Suggest that they focus initially on getting the mechanism to work, rather than on what it looks like. For reference, they can use the charts the class has developed, as well as the simple pop-up books they examined earlier in Lesson 1. If it is necessary to stop between periods, they can use the storage bags to keep work-in-progress.

**Suggested breakpoint between periods**

3. **Making pop-ups, continued** (Small groups – 20 min.): Students continue working on their pop-ups. As students are working, highlight their successes, by showing successful designs to the entire class. Meanwhile, collect a list of issues that they bring to your attention. Some of these might appear in their Notebooks. Others they might not write down, because they didn’t know how to solve them, or didn’t see them as issues. Look out for students who don’t ask for help, but nevertheless have serious difficulties in making a pop-up. You may need to work with them individually, or pair them with other students who can help them.

4. **Discussion of** **Troubleshooting** *Note: This could come at any point during the lesson – use your judgment to decide when it is most needed.* (Whole class – 20 min.): Observe that most of the pop-ups did not work immediately. Ask:

* If something doesn’t work right away, what should you do?

Help students develop the idea that they can figure out what’s wrong with something that doesn’t work. Most things that are faulty have only one thing wrong with them. It doesn’t make sense to start over again, because most of it actually does work! Also, if they start over, they might make the same mistake again. A much better procedure is to **troubleshoot**:

* Identify the issue that tells you that it isn’t working.
* Suggest a possible cause for that issue.
* Fix the cause that you think is creating the issue.

Post a chart with three columns labeled **Issue**, **Cause** and **Fix**, respectively, and fill in several rows, based on the issues that students have encountered. It is likely that some of the causes and fixes might already be in their science notebooks and/or on the Pop-up Principles chart.

**Science Notebook**:

1. **What you did:** Use pictures and the common language to describe how you tried to make a pop-up.

2. **Issues:** Things that didn’t work out the way you wanted.

3. **Troubleshooting**: How you tried to solve each problem, and how well it worked.

**5. Wrap-up** (Whole class 10 min.): After most students have created pop-ups, ask the class to wrap up. They can demonstrate their work, describe what they have learned and how they feel about making pop-ups.

## Word Bank

troubleshooting, hinge, link, linkage, gutter, page position, fold, paper engineer, issue, cause, fix

## 

# Lesson 3: The Shape of a Pop-up

## **Essential Question**

How does the shape of a pop-up change when you close and then open the book for the first time?

## **Task**

Learn to look at a pop-up from an edge view, and explore how its shape changes from this view

## Standards:

CCLS – ELA **Writing**: Text types and purposes  
**Speaking & Listening**: Comprehension and collaboration  
**Language**: Vocabulary acquisition and use

CCLS – Math   
**Math Practices**: MP2: Reason abstractly; MP7: Look for and make use of structure  
**Operations and Algebraic Thinking**: 4. OA5 & 5.OA3: Generate and analyze patterns and relationships  
**Geometry:** 5G:3 &4: Classify 2D shapes into categories

NGSS   
**Scientific & Engineering Practices** 1. Asking questions and defining problems; 3. Planning and carrying out investigations; 4. Analyzing and interpreting data; 8. Obtaining and evaluating information.  
**Crosscutting Concepts:** 1. Patterns; 2. Cause and effect: mechanism and prediction; 6. Structure and function; 7. Stability and change  
**Disciplinary Core Ideas**: PS2: Motion and stability: forces and interactions

## Outcomes

* Things look different from different points of view. Often, there is one viewpoint that offers the most interesting perspective.
* The shape changes when you close and open the book for the first time. A fold occurs in the pop-up piece, changing the shape from a “U” to a “V” from an edge view.
* Depending on where it is taped in, a pop-up will force the fold to be made in a particular place.

## Assessment

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Objective:** | **Below (1)** | **Approaching (2)** | **Proficient (3)** | **Advanced (4)** |
| A. Recognize that the shape changes based on both point of view and your action | Does not see an object from multiple points of view | Aware that shape changes, but unable to identify shapes; or does not view pop-up from an edge view | Identifies shapes seen before and after book is closed for the first time | Explains why shape changes after book is closed for the first time |
| B. Recognize that the new shape is determined by the structure | Not aware of location of fold | Aware that location of fold can change, but unaware of patterns in the location | Aware that location of fold is determined by structure | Aware that location of fold is determined by page positions |

## Advance Preparation

* Try the experiments yourself. These are demonstrated in the videos at <http://www.citytechnology.org/force-and-motion/the-shape-of-a-pop-up> , including the opening video and the videos under each step of Procedure.
* Prepare a few sample templates

## Materials

* Scissors and tape
* Ball-point pens and rulers, for making sharp folds. The pens don’t have to write.
* Parallel-fold Template, printed on 8 ½″ x 11″ cardstock, one per student
* Storage bags for saving pop-ups

## Procedure

**1. Using the template** (Whole class -- 15 min.) Provide each student with the Parallel-fold Template and ask them to cut out the six 12 cm. strips, along the dashed lines. If possible, provide each group with sample templates that you have already cut and taped. Note that the lines at 1 through 11 cm. are labeled, but not the two ends, at 0 and 12 cm., which are dashed. To make the book from the larger piece, fold it along the heavy center line to make the center line or gutter. See box below for tips on how to make a sharp fold. Make sure the vertical lines are inside the book – these will soon be used for measuring!

**How to make a fold**

1. **Score** the fold line by pressing hard along it, with a ballpoint pen, using a ruler as a guide to make sure the line is straight.
2. **Fold** the paper gently; it will naturally find the score line you have made.
3. Close the book, and **burnish** the fold by pressing along its length with a hard object, such as a scissors handle, fingernail or ruler.

**2. Assembling a pop-up** (Individual – 15 min.) Demonstrate a sample book with a strip taped in so it is not flat. Ask each student to make a similar book with just one of the strips taped onto the book. Remind them that it has to be taped on either side of the central fold. They should use the ruler lines to line up the edges of the strip, so its long sides are approximately **parallel** to the bottom and top edges of the book. The construction should look like Figure 1. They should not close the book just yet!



Figure 1: Book before closing

3. **Looking from an edge view** (Whole class – 10 min.) Once the books are assembled, ask students what they see:

* What shapes do you see?
* How do the shapes change if you look at it from another point of view?

Demonstrate how to look at something from an “edge view:”

1. Hold a single piece of ordinary paper with both hands so all you can see is the bottom edge. Close one eye. Instead of a rectangle, the sheet should look like a line.
2. Now do the same thing with the open book. The book itself looks like a straight line, and the pop-up piece should be a curved line above it. See Figure 2 a).

4. **Predicting and testing what it will look like** (Individual – 10 min.) Then challenge them to **predict** – but not yet **test** – what their books will look like from an edge view after they have closed and then opened it again.

They should not try the experiment until after they have made their predictions.

**Science Notebook**:

1. **Draw** how the book looks from an edge view **before** you close it.

2. Make a **drawing** showing your **prediction** of what the pop-up will look like **after** you close the book, and then open it again.

Once they have made their predictions, allow students to close and then open their books and see if their predictions were valid. Students may be surprised to find that the book makes a fold in the strip. Ask them to draw and describe the shape after closing and opening the book. See Figure 2 b).



Figure 2: Pop-up before and after closing

**Science Notebook**:

1. **Describe** how you did the experiment.

2. Use both pictures and words to show what the pop-up **looked like** after you closed the book, and then opened it again.

2. **Explain** why the pop-up looked the way it did.

Engage the class in a discussion of the predictions they made, what the results were, and why they came out the way they did.

**Suggested breakpoint between periods**

4. **Where** **does it make the fold**? (Individual – 15 min.) Ask students to tape two more strips into the book. One should be taped at the **same** **page positions** as the first strip, and the third should be taped at **different page positions**. Remind students that the page position is the distance from the gutter where the pop-up piece (strip) is taped into the book. The book has centimeter rulers printed in it, starting at the gutter, so it should be easy to see where the page positions are.

**Science Notebook**:

1. **Predict** where you think the folds will be in each of the three strips.

2. **Describe** what happened when you did the experiment.

3. **Explain** why you think the folds are where they are.

5. **Does the book respect the fold?** (Individual **–** 20 min**.)** Demonstrate another experiment. Remove all the previous strips from the book. Fold a new strip, and then tape it in, as in Figure 3. Ask students to predict what will happen to this strip after they close the book and open it again:



Figure 3: Strip is folded before closing the book.

**Science Notebook**:

1. **Draw** what the book looks like before you close it.

1. **Predict** what the folded strip will look like after you open the book.

2. **Describe** whether or not your prediction was correct.

3. **Explain** why the pop-up did or did not come out with another fold.

**6. Wrap-up** (Whole class – 10 min.) Lead a discussion about these experiments.

* What was surprising about what happened each time?
* What did you learn from each experiment?
* What other experiments would you like to try?

## Word bank

Score, burnish, prediction, test, results, point of view, edge view, arc, triangle

# Lesson 4: Finding Patterns in the Folds

## **Essential Question**

How do the page positions (places where the strips are taped in) affect the way a pop-up will look?

## **Task**

Do two controlled experiments that show the effect of using different page positions in the same book.

## Standards:

CCLS – ELA **Writing**: Text types and purposes; Research to build and present knowledge  
**Speaking & Listening**: Comprehension and collaboration  
**Language**: Vocabulary acquisition and use

CCLS – Math   
**Math Practices**: MP2: Reason abstractly; MP4: Model with mathematics; MP6: Attend to precision;   
MP7: Look for and make use of structure  
**Operations and Algebraic Thinking**: 4. OA5 & 5.OA3: Generate and analyze patterns and relationships  
**Geometry:** 5G:3 &4: Classify 2D shapes into categories

NGSS   
**Scientific & Engineering Practices** 1. Asking questions and defining problems; 3. Planning and carrying out investigations; 4. Analyzing and interpreting data; 7. Engaging in argument from evidence;   
8. Obtaining and evaluating information.  
**Crosscutting Concepts:** 1. Patterns; 2. Cause and effect: mechanism and prediction; 6. Structure and function

## Outcomes

* In a **controlled experiment** you change only one **independent variable** at a time. That way, you can find out how it affects the **dependent variable**. The other variables are **controlled**.
* In an **asymmetric** pop-up, the page positions affect where the fold will be.
* In a **symmetric** pop-up, the page positions affect the **height** of the pop-up

## Assessment

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Objective:** | **Below (1)** | **Approaching (2)** | **Proficient (3)** | **Advanced (4)** |
| A. Understand the need to control variables | Unaware of the need to control variables | Aware that variables should be controlled, but doesn’t explain why | Explains the need to control variables in a particular experiment | Explains the need to control variables in general |
| B. Recognize how page positions affect the location of the fold | Not aware of location of fold | Aware that one affects the other, but not of how each one varies | Aware that the fold is always on the far side, but doesn’t explain why | Explains why the book makes the fold on the far side |
| C. Recognize how page positions affect the height of the pop-up | Not aware of height of pop-up | Aware that one affects the other, but not of how each one varies | Aware that the height increases as the page positions move inwards, but doesn’t explain why | Explains why the height increases as the page positions move inwards, |

## Advance Preparation

* Do the two experiments, Near & Far and High & Low. These are demonstrated in the videos at <http://www.citytechnology.org/force-and-motion/finding-patterns-in-the-folds> , including the opening video on top and the videos under each step of Procedure. Save your work to show the class.
* Prepare two charts; one showing the page positions for each of the two experiments:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Near and Far** | | |  | **High and Low** | | |
| Pop-up | Page positions | | Pop-up | Page positions | |
|  | Left side | Right side |  | Left side | Right side |
| #1 | 1 cm. | 9 cm. | #1 | 2 cm. | 2 cm. |
| #2 | 3 cm | 7 cm. | #2 | 3 cm. | 3 cm. |
| #3 | 5 cm. | 5 cm. | #3 | 4 cm. | 4 cm. |
| #4 | 7 cm. | 3 cm. | #4 | 5 cm. | 5 cm. |
| #5 | 9 cm. | 1 cm. | #5 | 6 cm. | 6 cm. |

## Materials

* Scissors and tape
* Ball-point pens and rulers, for making sharp folds. The pens don’t have to write.
* Two Parallel-fold Templates per student, printed or photocopied onto 8 ½″ x 11″ cardstock.
* Science notebooks
* Storage bags for saving students’ pop-ups
* The books you created when you did the two experiments, Near & Far and High & Low.

## Procedure

**1. Near and far** (Whole class – 10 min.) Explain that we will be doing a **controlled experiment** to find out where the pop-up book “decides” to make its folds. There are five strips in the book, each taped in at different page positions. The page positions are the **independent variable** – we adjust them each time. We would like to know how these will affect the location of the fold in each strip, which is the **dependent variable**. Everything else stays the same for each strip – otherwise we wouldn’t know what caused the change in the dependent variable. The variables we keep the same are called **controlled variables**.

**2. Making the experiment book** (Individual – 20 min.) Post the chart you have made showing the page positions for Near & Far. Provide each student with the Parallel-fold Template and the Near and Far Worksheet. As in Lesson 3, students begin by cutting the six 12 cm. strips, and making the fold in the book. They then tape five strips at different page positions in the book, according to Table 1. The strips should not be folded first. As the assemble the pop-ups, they should label each one with its number, to keep track of them. They should not close the book yet.

**3. Predicting the outcome** (Whole class – 10 min.) After they have assembled the experiment , but before they close the book, ask students to predict the results of this experiment:

* What is the same about the way each strip is taped in? What is different about them?
* What pattern do they predict will happen in the folds?
* Why do they expect this pattern?

Chart their predictions.

**4. Doing the experiment** (Individual – 10 min.) Then ask students to conduct the experiment: simply close the book, open it again, and look for patterns in the folds. Worksheet 1 provides a format for writing their observations and conclusions. Discuss the results with the class.

* What pattern do you notice?
* How do the page positions “decide” where the pop-up will make the fold?
* Why do you think this happens?

**Suggested breakpoint between periods**

**5. High and low** (Whole class -- 15 min.) In doing the experiment Near & Far, students may have noticed that when the strip is centered, the fold line will come out in the center too. Introduce the concept of **symmetry**. Which (if any) of the pop-ups in the first experiment was **symmetric**?

* To make a symmetric pop-up, what do you have to do?

Once students have recognized that a symmetric pop-up would have the same page positions on each side (like Pop-up #3 in the previous experiment), explain that in this next experiment, all the pop-ups will be symmetric.

**6. Making the experiment book** (Individual – 20 min.) Post the chart you have made showing the page positions for High & Low. Provide each student with the Parallel-fold Template and the High & Low Worksheet. The procedure is the same as for Near & Far, except that the page positions are different.

**7. Predicting the outcome** (Whole class – 10 min.) After they have assembled the experiment , but before they close the book, ask students to predict the results of this experiment:

* What is the same about the way each strip is taped in? What is different about them?
* What pattern do they predict will happen in the folds?
* Why do they expect this pattern?

Chart their predictions.

**8. Doing the experiment** (Individual – 10 min.) Then ask students to conduct the experiment: simply close the book, open it again, and look for patterns in the folds. Worksheet 1 provides a format for writing their observations and conclusions. Discuss the results with the class.

* What pattern do you notice?
* What do the page positions “decide” about what the pop-up will look like?
* Why do you think this happens?

## Extension

Changing the variables Do additional experiments, using your own numbers for the page positions.

## Word bank

Controlled variable, Controlled experiment, Variable, Independent variable, Input variable, Dependent variable, Output variable Symmetry, Asymmetry, Height

## Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date: \_\_\_\_\_\_\_\_\_\_\_\_\_

**Near and Far**

1**. Set-up.** Tape five 12 cm. strips into the book at the page positions shown. **Do not fold the strips**. Label each strip with its number.

|  |  |  |
| --- | --- | --- |
| **Pop-up #** | **Page positions** (Distances from gutter) | |
| Left side | Right side |
| #1 | 1 cm. | 9 cm. |
| #2 | 3 cm | 7 cm. |
| #3 | 5 cm. | 5 cm. |
| #4 | 7 cm. | 3 cm. |
| #5 | 9 cm. | 1 cm. |

2. **The set up:** (before you close the book)

What is the same about each strip? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

What is different among the strips? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

What pattern do you expect among the strips after you close the book?

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Why do you expect this pattern? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

## Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date: \_\_\_\_\_\_\_\_\_\_\_\_\_

Near & Far, continued

3. **Results of the experiment:** Close the book, open it again, and look at it from an edge view. Draw two of the pop-ups from an edge view, one that is taped further to the left, and the other that is further to the right

Taped toward the left Taped toward the right

What is different between them?   
  
\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

What patterns do you notice about where the folds are made?

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Why do you think these patterns happen?

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

## Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date: \_\_\_\_\_\_\_\_\_\_\_\_\_

**High and Low**

1**. Set-up.** Tape five 12 cm. strips into the book at the page positions shown. **Do not fold the strips**. Label each strip with its number.

|  |  |  |
| --- | --- | --- |
| **Pop-up #** | **Page positions**  (Distances from gutter) | |
| Left side | Right side |
| #1 | 2 cm. | 2 cm. |
| #2 | 3 cm. | 3 cm. |
| #3 | 4 cm. | 4 cm. |
| #4 | 5 cm. | 5 cm. |
| #5 | 6 cm. | 6 cm. |

2. **The set up:** (before you close the book)

What is the same about each strip? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

What is different among the strips? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

What pattern do you expect among the strips after you close the book?

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Why do you expect this pattern? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

## Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date: \_\_\_\_\_\_\_\_\_\_\_\_\_

High & Low, continued

3. **Results of the experiment:** Close the book, open it again, and look at it from an edge view. Draw two of the pop-ups, one that is taped closer to the gutter and the other that is taped further from the gutter:

Closer to the gutter Further from the gutter

What is different between them?   
  
\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

What patterns do you notice about where the folds are made?

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Why do you think these patterns happen?

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

# Lesson 5: The Pop-up Computer

## **Essential Question**

Where does the pop-up book make the fold?

## **Task**

Collect and analyze data showing how the pop-up book divides up the taped-in strip when it makes the fold.

## Standards:

CCLS – ELA **Speaking & Listening**: Presentation of knowledge and ideas  
**Language**: Vocabulary acquisition and use

CCLS – Math   
**Math Practices**: MP2: Reason abstractly; MP3: Construct viable arguments; MP4: Model with mathematics; MP6: Attend to precision; MP7: Look for and make use of structure  
**Measurement & Data**: 4. MD4: Represent and interpret data

NGSS  
**Scientific & Engineering Practices** 2. Developing and using models; 3. Planning and carrying out investigations; 4. Analyzing and interpreting data; 7. Engaging in argument from evidence; 8. Obtaining and evaluating information.  
**Crosscutting Concepts:** 1. Patterns; 6. Structure and function

## Outcomes

* If an experiment includes measurement, the collected data can analyzed to find hidden patterns;
* Data analysis can lead to symbolic representations, which can make it easier to express the patterns;
* A pop-up book follows definite rules when it makes the fold; these rules can be discovered through data collection and analysis, and represented symbolically.

## Assessment

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Objective:** | **Below (1)** | **Approaching (2)** | **Proficient (3)** | **Advanced (4)** |
| A. Measure lengths accurately | No measure-ments | Makes measurements that are inconsistent or inaccurate | Makes accurate measurements | Identifies and Corrects inaccurate measurements made by self or others |
| B. Finds patterns in the data and represents them symbolically | No patterns found | Finds pattern, but does not express it clearly | Expresses pattern in words | Expresses patterns in words and symbols |

## Advance Preparation

* Do the basic experiment yourself. Collect several rows of data and analyze them. The method is demonstrated in the videos at <http://citytechnology.org/force-and-motion/the-pop-up-computer> , including the opening video on top and the videos under each step of Procedure. Save your work to show the class.
* Prepare three charts: a) one showing the links and hinges in a pop-up (see Figure 1); b) another showing the four link lengths (Figure 2); and c) a chart for collecting data (Figure 3).



Figure 1: The four hinges and the four links of a simple pop-up (these can be combined into one chart, using different colors for the hinges and links)

Figure 2: The four links, A, B, C and D, and how their lengths are measured (these can be combined into one chart, using different colors for the letters and the arrows)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Class Data Table | | | | | |
| Name | A | B | C | D | Notes |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

Figure 3: Table for collecting and analyzing data from the whole class

## Materials

* **Scissors, tape, ball-point pens, markers and rulers**
* **Parallel-fold Template,** printed or photocopied onto 8 ½" x 11" cardstock.
* **Science notebooks**
* **Storage bags** containing previous work from Lessons 3 & 4.

## Procedure

1. **The Pop-up Computer** (Whole class – 10 min.) Review the previous two lessons:

* What did we learn for the experiments about how pop-ups work?

Guide the discussion toward the discovery that a pop-up book always seems to “know” where it “wants” to make the fold. This lesson will reveal how it does that.

2. **Links and Hinges** (Whole class – 15 min.) Ask what these terms mean:

* What does a **hinge** do?
* Where can you find one?

Student may identify hinges on the classroom door, cabinet doors, and folding furniture. Other examples of hinges are the spine of a book, a knee, the pivot in a scissors, etc. Relate these to pop-ups:

* How does the word hinge relate to a pop-up?
* Take out one of the pop-ups you have made and mark the hinges.
* How many hinges does it have?

Explain that the rigid pieces that rotate around the hinges are called **links**:

* Where are the links in a pop-up? How many links does it have?

Show the class the chart in Figure 1. A simple pop-up mechanism has four hinges and four links.

**3. Measuring link lengths** (Small groups **--**  25 min.) Show the class the chart in Figure 2. Explain that the four links are labeled A, B, C and D. Their lengths are also labeled A, B, C and D. Each of these lengths is the distance between two hinges. To get the pop-up to reveal its secrets, the next task will be to measure and record these four distances, A, B, C and D.

Distribute templates, and ask each students to cut a strip and tape it into the book. Then close the book, open it again, and label the four links, A, B, C and D.

To measure A, C and D, they can just read the rulers printed in the book and on the strip. A and D show the distance from the gutter in the book, and C shows the distance from the left end of the strip – the page position – to the fold. However, B is a little bit harder, because the distance must be read from the fold line to the other end of the strip, which is not represented directly, and requires subtraction. If students have difficulty with this problem, you can use the following optional mini-lesson. Either way. provide time for students to practice measuring.

**Mini-Lesson for measuring link lengths on the strip**

Cut a strip from the Template.

* How long is it?
* If this strip was taped into the book, which link lengths would it contain?
* What will these link lengths have to add up to?

Now fold the strip at the line numbered 5. Unfold it and mark the fold line with a marker. Then,

* + Label the two link lengths that would be on the strip if it were taped into the book.

Figure 4 shows what it will look like.



Figure 4: Strip folded at 5 cm., with link lengths B and C labeled

What are the link lengths B & C?

Many students will say that both B & C are 5 cm., because that’s what the number says at the fold line. They couldn’t be, because B and C have to add up to the total strip length, which is 12 cm., not 5 + 5 = 10 cm. C is indeed 5 cm., because the numbers start at 0 on the left end.

But what is B? Help students see that B + C = 12 cm. We already know that C is 5 cm., so you can find B by subtracting this number from 12: B = 12 – 5 cm. = 7 cm.

**Suggested breakpoint between periods**

**4. Collecting and recording data** (Small groups – 30 min.)Once students are comfortable with making the measurements, they are ready to do the full experiment.Ask each student to make four pop-ups in the same book, using different page positions for each one. They should number the pop-ups 1, 2, 3 & 4, and label the links A, B, C & D in each pop-up. Then they should measure four link lengths for each of the four pop-ups – a total of 16 measurements. Provide worksheets for them to record the data. Other group members should check each student’s data.

**5. Class discussion of the data** (Whole class – 20 min.)Post the ClassData Table (Figure 3)**.** Ask a student from each group to provide a single row of data from their worksheet. Write the data on a row of the table, identified by the student’s name. Discuss the data that has been entered:

* + What pattern do you see?
  + Look at each row? Is it good data? How can you tell?
  + If it’s not good data, what could have gone wrong?

## Extensions

## 1. How can you make it lie flat or stand up?

## Make several pop-ups that lie flat when the book is open. Measure and record A, B, C and D for each one. What pattern do you find in the data?

2. What happens if you break the rule?

Fold the strip before taping it in, and then tape it in such a way that   
A + B does not = C + D. Explore what happens when you then try to force the book to close.

## Word Bank

## measurement, length, hinge, link, data collection, data table, data analysis, pattern, hypothesis, equation, symbolic representation Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date: \_\_\_\_\_\_\_\_\_\_\_\_\_

The Pop-up Computer

**A. Data Table**: Select four pop-ups you have made using the Template. Label each pop-up with a number. Measure the four link lengths and enter the data in the table.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Pop-up # | A | B | C | D | Observations |
| 1 |  |  |  |  |  |
| 2 |  |  |  |  |  |
| 3 |  |  |  |  |  |
| 4 |  |  |  |  |  |

## Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date: \_\_\_\_\_\_\_\_\_\_\_\_\_

The Pop-up Computer, continued

**B.** **Class Data Table**. After your class has shared its data, fill in A, B, C and D from other students. Use only data that your teacher described as “good data.” Then look for patterns in the data. To do this, fill in the two new columns labeled A + B and C + D

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| A | B | C | D | A + B | C + D | Observations |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |

**C. Patterns in the data:** What pattern do you notice in this data?

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**D. Using Symbols**: Write an equation that expresses this pattern: \_\_\_\_\_\_\_\_\_\_\_

# Lesson 6: Where does it Hide?

## **Essential Question**

How does the pop-up book make sure that A + B always = C + D?

## **Task**

Explore what happens to the strip when the book is closed, by examining it inside the “See-thru Book”?

## Standards:

CCLS – ELA **Writing**: Text types and purposes; Research to build and present knowledge  
**Speaking & Listening**: Comprehension and collaboration  
**Language**: Vocabulary acquisition and use

CCLS – Math   
**Math Practices**: MP2: Reason abstractly; MP3: Construct viable arguments; MP4: Model with mathematics; MP6: Attend to precision; MP7: Look for and make use of structure  
**Operations and Algebraic Thinking**: 4. OA5 & 5.OA3: Generate and analyze patterns and relationships  
**Measurement & Data**: 4. MD4: Represent and interpret data

NGSS  
**Scientific & Engineering Practices** 1. Asking questions and defining problems; 4. Analyzing and interpreting data; 3. Planning and carrying out investigations; 4. Systems and system models;   
6. Constructing explanations;7. Engaging in argument from evidence; 8. Obtaining and evaluating information.  
**Crosscutting Concepts:** 1. Patterns; 2. Cause and effect: mechanism and prediction; 4. Systems and system models; 6. Structure and function  
**Disciplinary Core Ideas:** PS2: Motion and Stability

## Outcomes

* By looking deeper, it is often possible to find out why things happen. This can provide an **explanation** of why one thing **causes** another.
* The see-thru book makes it possible to see what happens to the links when the book is closed, which explains why A + B has to equal C + D.

## Assessment

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Objective:** | **Below (1)** | **Approaching (2)** | **Proficient (3)** | **Advanced (4)** |
| A. Finds patterns in link lengths using the see-thru book | No patterns found | Finds patterns that are incomplete or inconsistent | Finds patterns accurately | Uses patterns to explain additional facts, such as why the pop-up stays in or sticks out of the book |
| B. Use results of the experiment to explain why A + B = C + D | No explanation | Finds pattern, but does not express it clearly | Expresses pattern in words | Expresses patterns in words and symbols |

## Advance Preparation

* Do the experiment yourself. Visit <http://citytechnology.org/force-and-motion/where-does-it-hide> to see four short videos that will walk you through the experiment.
* Prepare a few sample see-thru books, with the experiment inside, to wshowcstudents in case they have difficulty

## Materials

* See-thru Book
* Science Notebooks
* Scissors, tape, ball-point pens and rulers
* Parallel-Fold Template

## Procedure

1. **When does it make A + B = C + D?** (Whole class – 10 min.) Review what happened in Lesson 5: a pop-up seems to make the rule A + B = C + D. Ask the class:

* When does the pop-up book make this happen? How? Why?
* If you started out with link lengths that did not follow this rule, what would the book do to your strip? When?

Students should come to the conclusion that the book forces A + B = C + D to happen when it closes. To understand the rule, it will help to know what happens inside the book when it closes.

2. **Predicting where it will hide** (Individual – 10 min.) Distribute the Worksheet. The picture shows three different pop-ups, with the pop-up piece in different positions. In the middle column, ask students to predict where each of these pop-ups will “hide” when the book is closed. They do so by drawing the shape and location of the pop-up piece inside the closed book. Once they have done so, ask:

* How can we find out if our predictions are correct?

Point out that you can’t see inside the closed book, because it is **opaque** (unless you are Superman!**)**. What if we had a book that was **transparent**, so anybody could see through it?

3. **The see-thru book** (Individual – 30 min.) Provide each student with a See-thru book. Their task is to assemble each of the three pop-ups inside the See-thru book, close it, and look to see if the pop-up piece is where they predicted it would be on the Worksheet.

**Science Notebook**:

**Discuss** your results. What happened? Did the results come out the way you expected? Why or why not?   
What did you learn from this experiment?

**Suggested breakpoint between periods**

4. **Explaining where it hides** (Whole-class – 20 min.) Lead a discussion based on what students have written in their notebooks.Provide time for students to try to figure out for themselves why the pop-uphides where it does. Here are some scaffolds for helping them figure this out.

* What does each page of the book do as the book is closed?
* Where is the gutter after the book is closed?
* What does each page position line do as the book is closed?
* What direction does the pop-up fold line travel as the book closes?
* What will happen to the four links, A, B, C & D, as the book closes?

**Science Notebook**:

1. Which way does the pop-up move when you close the book? Why?

2. What is making it move? What parts of it can’t move and what parts can?

**5. Why does A + B = C + D?** (Individual – 20 min.) Ask students to label link lengths A, B, C and D inside the see-thru book. Lengths A and D can be written on masking tape, and attached to the book.

* Look at the locations of A, B, C and D when the book is closed. Which ones are on the bottom? Which ones are on top? Make a drawing showing A, B, C and D.
* Where do A and B start and stop? Where do C and D start and stop?
* If two lines lie one on top of the other, and start and stop in the same place, what do we know about them?

**Science Notebook**:

1. **Discuss** the results. What did the clear folder show about the relationship among A, B, C & D?

2. **Explain** the results. Why do you think A + B = C + D? Do you think this will always work? Why or why not?

## Word Bank

Cause, effect, explanation, opaque, transparent, point of view, top view, edge view, rhombus, quadrilateral, rectangle, square

## Extensions

1. How can you make it stay inside the book?

Using the see-thru book, make some pop-ups that stay inside the book when the book is closed; others that stick out, and one that just touches the edge. Notice and draw the link lengths in each case. What relationships have to be true for a pop-up to:

1. Stay inside the book?
2. Stick out of the book?
3. Just touch the edge of the book?

2. Geometry through pop-ups

1. Make an **asymmetrical** pop-up that lies flat when the book is completely open (see Extension 1, Lesson 5). Open it slowly while you look at it from an edge view. What shapes do you see as the book goes from fully closed to fully open? Why does it make these shapes?
2. Make a **symmetrical** pop-up that lies flat when the book is fully open. This requires the two page positions, A and D to be equal. Open it slowly while you look at it from an edge view. What shapes do you see as the book goes from fully closed to fully open? Why does it make these shapes?

## Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date: \_\_\_\_\_\_\_\_\_\_\_\_\_

Where does it Hide?

1. **Examine** the pop-up in each row on the left. It is assembled so the pop-up piece lies flat when the book is open.
2. **Imagine** that the book is closed by turning the page on the left side.
3. In each row, **predict** where the pop-up piece will hide inside the book when it is closed. Draw your prediction in the middle rectangle.
4. **Check** each prediction by making the pop-up inside the see-thru book.
5. In the column on the right, **draw** where it actually went.



Lesson 7: Stair-step Pop-ups

## **Essential Question**

How can one pop-up be built on top of another?

## **Task**

Create pop-ups that build upon one another inside the same book.

## Standards:

CCLS – ELA **Writing**: Research to build and present knowledge  
**Speaking & Listening**: Comprehension and collaboration  
**Language**: Vocabulary acquisition and use

CCLS – Math   
**Math Practices**: MP3: Construct viable arguments; MP6: Attend to precision; MP7: Look for and make use of structure  
**Geometry:** 5. G2&3: Classify 2D shapes

NGSS  
**Scientific & Engineering Practices** 2. Developing and using models; 3. Planning and carrying out investigations; 4. Systems and system models; 6. Designing solutions; 7. Engaging in argument from evidence; 8. Obtaining and evaluating information.  
**Crosscutting Concepts:** 1. Patterns; 2. Cause and effect: mechanism and prediction; 4. Systems and system models; 6. Structure and function**;** 7. Stability and change **Disciplinary Core Ideas:** PS2: Motion and stability

## Outcomes

* A system has an input and an output. A pop-up is a system whose input is opening or closing the book, and whose output is the resulting motion of the pop-up.
* By building one pop-up on top of another, you create a complex system where each pop-up is a subsystem.
* Pop-ups built on one another can make **similar shapes** from an edge view.

## Assessment

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Objective:** | **Below (1)** | **Approaching (2)** | **Proficient (3)** | **Advanced (4)** |
| A. Identify systems, inputs and outputs | No inputs or outputs identified | Inputs or outputs are identified inconsistently | Identifies systems, inputs and outputs reliably | Draws analogies between different kinds of systems |
| B. Describe and diagram compound systems | No description or diagrams | Not aware of both compound systems, and subsystems | Describes and diagrams compound systems and subsystems | Distinguishes between series and parallel systems |
| C. Recognize similar shapes from an edge view as a pop-up opens and closes | No shapes found | Recognizes shapes but not their similarity | Recognizes similar shapes | Finds similar shapes in multiple systems |

## Advance Preparation

* Create some stair-step pop-ups. For guidance, view the opening video and videos under Procedure, steps 2 & 3 at <http://citytechnology.org/force-and-motion/stair-step-pop-ups>.
* Prepare two charts: a) a chart for tabulating examples of systems, with three columns labeled System, Input and Output, and b) a diagram showing construction of a stairs-step pop-up, like in Figure 1:  
    
  



Figure 1: Steps in Assembling a Stair-step Pop-up: Top: 1st pop-up; Bottom: 2nd pop-up built on 1st

## Materials

* Scissors, tape, ball-point pens and rulers
* Parallel-Fold Template
* Storage bags for saving pop-ups
* Science notebooks

## Procedure

1. **Systems** (Whole class / Individual / whole class – 20 min.) Develop the most basic systems concepts**: input, output** and **system**. Ask students to complete the Systems Worksheet. In Part A, they identify the inputs and outputs of a video game, and then they add some of their own choice. In Part B, they label the inputs and outputs on a system diagram. After students have completed their worksheets, conduct a whole-class discussion in which students provide examples from their worksheets, and discuss each example with the class. In identifying the inputs and outputs, focus on the **actions** that happen at the inputs and outputs. After there is a consensus, add the examples to the class chart. Then add Pop-up on the bottom line under the System column, and ask students to identify the input and output to a pop-up. Develop the conclusion that the input is the opening or closing of the book, and the output is that something pops up.

2. **The Stair-step Pop-up** (Individual – 30 min.) Ask students:

* How can one pop-up be built on top of another?

If they don’t come up with the idea themselves, present the Stair-step Pop-up chart you have made. Suggest that students begin by making a simple pop-up, with only one pop-up piece, using the same technique they have used before. The strip should be taped in flat, before closing the book, as in the Top part of Figure 1. As you open the book, pull up gently on the pop-up to make sure it folds outward. The next step is to add another pop-up to the first one, placing it across the right page position, as in the Bottom of Figure 1.Again, make sure the pop-up folds outward as the book is opened. Provide time for students to experiment with this construction.

**Suggested breakpoint between periods**

**3. Fractions and Angles**  (Whole class -- 15 minutes) Review basic **fractions**, as they might be applied to the opening angle of a book::

* When is a book fully open? When is it ½-open? ¼-open? ¾-open?

Review **angles** in the same context:

* What does it mean to say that an angle is **acute**? **obtuse**? A **right angle**?
* Look at the angle between the covers of a book, as it changes from closed, to ¼-open, ½-open, ¾-open and finally fully open. When is the angle between covers acute? When is it obtuse? A right angle?

**4. Edge views**: (Individual – 15 min.) Ask students to look at their new pop-ups from an edge view when it is:

1. ¾ -open b) ½-open c) ¼-open

They can use the Stair-step Pop-up Worksheet, Part 1, to draw what they see, and then conduct a brief discussion:

* What do they notice about these shapes?
* What is similar between the shapes made by the 1st and 2nd pop-up in each diagram?
* Why do we call this a Stair-step pop-up?

Develop the concept of **similar shapes**, which have the same shape, but different sizes,like similar triangles.

**5. System diagrams** (Whole class / Individual – 20 min.) Provide some prompts for scaffolding the creation of a **system diagram** for a stair-step pop-up:

* What is the input to the 1st pop-up?
* What is the input to the 2nd pop? (What does it “think” is its “book”?)
* If the 1st pop-up wasn’t there, what would happen to the 2nd one?

Develop the idea that the output of the 1st pop-up is the input to the 2nd. Discuss how this could be shown in a system diagram. Introduce some words: the stair-step construction actually includes two pop-ups, which are systems in themselves. The overall system is a **compound system**, and the two individual pop-ups are its **subsystems**. Because the output of one is the input to another, they are in a **series connection**. Using this information, students can try to construct a System Diagram for the Stair-step Pop-up in Part 2 of the worksheet.

## Word Bank

system, system diagram, compound system, subsystem, fraction, input, output, parallel connection, series connection, acute angle, right angle, obtuse angle, similar shapes

## Extensions

1. Add more pop-ups:

Look through commercially made pop-ups to find more than one pop-up between the same pages. How are they connected? Make a construction that has more than two pop-ups within the same folder. How are they connected?

2. System diagrams for more complex pop-ups

Create a system diagram for each of the constructions made or examined in Extension 1.

## Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date: \_\_\_\_\_\_\_\_\_\_\_\_\_

Worksheet 1: **Inputs, Outputs and Systems**

Key

**Input = Action you take to make it work**

**Output = Result you look for due to your action**

A. In Row 1, identify the input and output of a video game; then add 6 more systems and the input and output of each one

|  |  |  |  |
| --- | --- | --- | --- |
|  | **System** | **Input** | **Output** |
| 1 | Video game |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |

B. The drawing below shows a System Diagram. Label the arrows and box with the terms that fit: **Input**, **Output** and **System**.



## Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date: \_\_\_\_\_\_\_\_\_\_\_\_\_

Worksheet 2: **The Stair Step Pop-up**

1. **Edge views**: What do you notice about the new pop-up you have made ?

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Draw in all the inside parts from an edge view in the ¾-, ½- and ¼-open positions:



What do you notice about these shapes? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**2. System diagram:** In the space below, draw a system diagram for a stair-step pop-up.

# Lesson 8: How they Move

## **Essential Question**

What kinds of motion can a pop-up produce?

## **Task**

Analyze pop-ups to find out different kinds of motion

## Standards:

CCLS – ELA **Writing**: Text types and purposes; Research to build and present knowledge  
**Speaking & Listening**: Comprehension and collaboration  
**Language**: Vocabulary acquisition and use

CCLS – Math   
**Math Practices**: MP3: Construct viable arguments; MP6: Attend to precision; MP7: Look for and make use of structure  
**Operations and Algebraic Thinking:** 4. OA5 & 5. OA3: Generate and analyze patterns and relationships **Geometry:** 4. & 5. G1,2&3: Draw and identify lines and angles

NGSS  
**Scientific & Engineering Practices** 1. Asking questions and defining problems; 3. Planning and carrying out investigations; 4. Systems and system models; 6. Constructing explanations; 7. Engaging in argument from evidence; 8. Obtaining and evaluating information.  
**Crosscutting Concepts:** 1. Patterns; 2. Cause and effect: mechanism and prediction; 6. Structure and function**;   
Disciplinary Core Ideas:** PS2: Motion and stability

## Outcomes

* Simple pop-ups come in two main categories. One type produces horizontal but not vertical motion. The other type produces both horizontal and vertical motion
* In two **dimensions**, lines must either **intersect** or be **parallel**. In three dimensions, there are also **skew lines**, which neither intersect nor are parallel.
* The type of motion of a pop-up depends on the hinge lines. If the motion is only horizontal, the hinge lines must be parallel. If it is both horizontal and vertical, the hinge lines must intersect at a point, called the **vertex**.

## Assessment

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Objective:** | **Below (1)** | **Approaching (2)** | **Proficient (3)** | **Advanced (4)** |
| A. Sort pop-ups based on their directions of motion | Directions of motion are not identified | Directions of motion are found inconsistently | Two types of pop-up are identifed | Recognition that there are additional types that do not fall into either category |
| B. Identify the possible relationships between two lines | Relationships are not identified | Identifies parallel and intersecting lines on 2D plane | Recognizes parallel, intersecting and skew lines in 3D space | Explains why there are no skew lines on 2D plane |
| C. Recognize how the relationship between the hinge lines governs the directions of motion | No connection found | Recognizes relationship between hinge lines, but not how it affects motion | Recognizes connection between hinge lines and directions of motion | Explains why hinge lines cannot be skew |

## Advance Preparation

* Try the activity yourself. For guidance, view the opening video and videos under Procedure at <http://citytechnology.org/force-and-motion/how-they-move>.
* Create the chart shown below; leave space for the last two columns on the right and one more row at the bottom

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Dimensions and Directions of Motion** | | | | |
| Dimension | Directions | Description |  |  |
| Horizontal | back ↔ forth | Across the gutter |  |  |
| Vertical | up ↕ down | Along the gutter |  |  |
| 3D | in ↔ out | Towards or away from the book |  |  |
|  |  |  |  |  |

Table 1: Dimensions and directions of pop-up motion

## Materials

* Assorted pop-up books and cards, including pop-ups already made by students, simple pop-ups in commercially made books and cards, and intermediate pop-ups.
* Post-its ™ and scissors for making flags
* Science notebooks

## Procedure

**1. Exploring pop-up motion** (Whole class/ small groups – 20 min.). Distribute sample pop-ups, including commercially made books and cards, as well as pop-ups made by students. Make sure that each group has some pop-ups that include angle-folds (Type B), as well as others that use parallel-folds (Type A). These terms are defined below. Students will be looking at how the fold line moves as the book opens and closes. Show students how to make flags from a Post-It™ (Figure 1) and attach one to each pop-up along the fold line, as in Figure 2.



Figure 1: Cutting two strips from a Post-It™ for use as flags



Figure 2: Attaching a flag

Provide time for students to look at how the flags move in various types of pop-ups.

**2. Examining pop-ups for type of motion** (Small groups – 10 min.). Present the chart, Dimensions and Directions of Motion. Ask students to show with their hands what these terms mean. Then demonstrate a standard way of looking at a book: hold it vertically, with one page in each hand. Open the book by moving the left side only, holding the right side fixed. It should look like in Figure 3 as the book is opened. Then ask:

* Which way(s) can your pop-ups move?
* Look at all your pop-ups to see if they all can make the same kinds of motion. If not, sort them into categories according to the motion they can do.

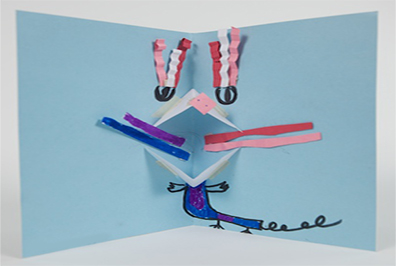
 

Figure 3: Opening a book a standard way

**3. What categories are there?** (Whole class – 10 min.) Ask the class to share what they found. If necessary, demonstrate two pop-ups with different kinds of motion:

* Type A, which can move horizontally (back and forth), plus 3D (in and out), but not vertically;
* Type B, which can move in all three dimensions.

Mention that paper engineers call Type A the **Parallel-fold pop-up**, and Type B the **Angle-Fold pop-up**. Nearly every pop-up in the world is made from combinations of these two types! Summarize all this by filling in the last two columns of the chart:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Dimensions and Directions of Motion** | | | | |
| Dimension | Directions | Description | A: Parallel-fold | B: Angle-fold |
| Horizontal | back ↔ forth | Across the gutter | yes | yes |
| Vertical | up ↕ down | Along the gutter | no | yes |
| 3D | in ↔ out | Towards or away from the book | yes | yes |

**4. Looking again at pop-ups** (Small groups – 10 min.) Once the categories have been established, ask each group to sort all of their pop-ups according to the two types. Keep these pop-ups in their categories for the next sequence of activities.

**Science Notebook:**

What similarities can you find about the pop-ups in each category, which might **explain** the different directions of motion?

**Suggested breakpoint between periods**

**5. Lines in space** (Whole class – 20 min.) Using a cardboard box or a large book, ask how lines in space can be related:

* What happens to lines that are not parallel?
* How could two lines be neither parallel nor intersecting?

Develop the idea that lines on a plane have to either intersect or be parallel, but in 3d space they could be neither. Lines that do not intersect and are also not parallel are called **skew** lines.

**6. Sorting pop-ups according to type of construction** (Small groups – 20 min.) Review the four **hinges** of a pop-up (Lesson 5). Each hinge follows a line. Ask students:

* How do the four hinge lines run in each type of pop-up?
* Look at all the pop-ups in the category to see if the same rule applies to all of them.

**Science Notebook**

1. **Discuss** what was different between the hinge lines in the two types of pop-ups.

2. **Explain** why you think the different kinds of hinge lines lead to different kinds of motion.

**7. What was learned** (Whole-class – 10 min.) Ask the groups to report on what they have found. They should have observed that:

* In a Type A pop-up (Parallel-fold), all the hinge lines run parallel
* In a Type B pop-up (Angle-fold), all the hinge lines intersect at a point.

Summarize all of this by filling in the bottom row of the table:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Dimensions and Directions of Motion** | | | | |
| Dimension | Directions | Description | A: Parallel-fold | B: Angle-fold |
| Horizontal | back ↔ forth | Across the gutter | yes | yes |
| Vertical | up ↕ down | Along the gutter | no | yes |
| 3D | in ↔ out | Towards or away from the book | yes | Yes |
| **Hinge lines** |  |  | **parallel** | **intersecting** |

The point where the hinge lines intersect is called the **vertex**.

## Word bank

angle-fold pop-up, axis, direction of motion, flag, horizontal, vertical, dimension, two-dimensional, three-dimensional, plane, space, force component, intersecting lines, parallel lines, parallel-fold pop-up, perpendicular, skew lines, vertex

## Extension

A pop-up whose hinge lines are skew: Try making a pop-up whose hinge lines are neither parallel nor intersecting – in other words, which are skew lines. What happens when you try to close it?

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date: \_\_\_\_\_\_\_\_\_\_\_\_\_

**Two Types of Pop-up**

For each type of pop-up, show how the pop-up will move and draw the four hinge lines: gutter, fold & 2 page positions.

|  |  |  |
| --- | --- | --- |
| **Type** | **Show the direction  the output will move in** | **Draw the hinge lines** |
| A |  |  |
| B |  |  |

**Type A:** If you want to make a pop-up that can only move back & forth and in & out, but not up & down, how should you make your hinge lines?

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Type B:** If you want to make a pop-up that can move up & down as well as back & forth, how should you make your hinge lines?

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

# Lesson 9: Make an Angle-fold Pop-up

## **Essential Question**

How can you make a pop-up that has both vertical and horizontal motion?

## **Task**

Create angle-fold pop-ups

## Standards:

CCLS – ELA **Writing**: Text types and purposes; Research to build and present knowledge  
**Speaking & Listening**: Comprehension and collaboration  
**Language**: Vocabulary acquisition and use

CCLS – Math   
**Math Practices**: MP4: Model with mathematics; MP6: Attend to precision; MP7: Look for and make use of structure  
**Operations and Algebraic Thinking:** 4. OA5 & 5. OA3: Generate and analyze patterns and relationships **Measurement and Data:** 4.MD5, 6 & 7: Understand concepts of angle and measure angles **Geometry:** 4 & 5. G1,2&3: Draw and identify lines and angles

NGSS  
**Scientific & Engineering Practices** 1. Asking questions and defining problems; 3. Planning and carrying out investigations; 4. Analyzing and interpreting data; 6. Designing solutions; 7. Engaging in argument from evidence; 8. Obtaining and evaluating information.  
**Crosscutting Concepts:** 1. Patterns; 2. Cause and effect: mechanism and prediction; 6. Structure and function**;   
Disciplinary Core Ideas:** ETS1: Engineering Design

## Outcomes

* The amount of motion changes depending on where you put the flag. The most motion happens where the fold line crosses the flat side of the triangle, opposite the vertex. You can also get more motion by using a longer flag.
* Two angle folds in the same book will move away from each other or towards each other, depending on the triangles are pointing inwards or outwards.

## Assessment

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Objective:** | **Below (1)** | **Approaching (2)** | **Proficient (3)** | **Advanced (4)** |
| A. Explore the amount of motion at different points on an angle fold | No exploration | Recognizes that the amount of motion can change, but doesn’t know how | Finds where the amount of motion is maximum | Explains why motion is maximum and uses this fact in designing a pop-up |
| B. Explore the motion of two angle folds in the same book, facing towards or away from each other | No exploration | Recognizes that they move differently | Identifies motion of each one | Explains why they move differently and designs pop-ups that use these motions |

## Advance Preparation

* Make a basic Angle-fold, a Monster and a Mouth. For guidance, view the opening video and videos under Procedure at <http://citytechnology.org/force-and-motion/make-an-angle-fold-pop-up> and at <http://citytechnology.org/force-and-motion/the-monster-and-the-inside-out-monster>.
* Make three charts: a) Angle-fold Dos and Don’ts (Figure 1); b) The Monster construction (Figure 2)   
   and c) Reversing the Triangles (Figure 3)



Figure 1: Angle-fold Dos and Don’ts



Figure 2: The monster



Figure 3: Reversing the Triangles

## Materials

* Angle-fold Template, two per student
* Scissors, tape, rulers, ball-point pens and Post-Its ™
* Science notebooks
* Storage bags for saving pop-ups, at least one per group

## Procedure

1. **The Basic Angle Fold** (Whole class/ Individual – 30 min.) Tell the students that they will finally have a chance to make “Type B” pop-ups – ones that produce vertical as well as horizontal motion. The Angle-fold Template will make it easy to make these. Post the chart showing Angle-Fold Dos and Don’ts, and review the instructions. To begin with, they will need only one of the triangles – they can save the rest for later.

**2. Where can you get the most motion?** (Small group – 10 min.) Ask:

* What part of the angle-fold moves the most?

Ask students to predict the answer, and then test their predictions by moving the flag to different points on the triangle.

**Science Notebook**:

Which part of an angle fold moves the most? Which part moves the least? Why do you think so?

**3. Assemble it unfolded**: (Individual – 10 min.) The angle-fold made previously will lie flat when the book is open. To make it work differently, take a triangle that has not been folded. Push it in on both sides so it makes a little arch, and then tape it on both sides. Make sure the vertex touches the gutter. Predict what will happen when you close the book.

**Science Notebook**:

**Predict** what youthink will happen to it when you close the book.

Now allow students to close their books, pressing hard along the gutter to get a good fold. When they open their books, they should look inside to see what happened.

**Science Notebook**:

**Describe** what the pop-up looks like. Was your prediction correct?

**Explain** how your new pop-up is different from the first one you made.

**Suggested breakpoint between periods**

4. **The Monster** (Individual –20 min.) Post the chart showing how to make the Monster. Tell students to tape both triangles in the positions indicated on the Angle-Fold template.

**Science Notebook**:

**Predict** what youthink the pop-up will do as you close and open the book.

After making the prediction, they should try it to see what it does.

**Science Notebook**:

1. **Describe** what the pop-up looks like. Was your prediction correct?

2. **Explain** how you could use this idea for your own design. Why do you think we call it “The Monster”?

Provide time for students to experiment with this construction. It can produce very dramatic motion, as the two flags fling in opposite directions. We call it the **Monster,** becauseone student attached the monster’s head to the angle-fold on top, the tail to the one on the bottom, and drew the middle of the monster directly on the book.

**5.** **Direction of motion of the Monster** (Whole class – 10 min.) Ask students to observe the motion of the Monster pop-up. Lead a discussion about these issues:

* Which part of the triangle is moving when the book opens or closes?
* Which part is not moving?
* What would change if the angle-fold on the top were moved to the bottom, and vice versa?

**6. Reverse the triangles** (Individual – 20 min.)Show the third chart, Reversing the Triangles.Ask:

* What will happen if you turn each triangle around, so the vertices are pointing away from each other, instead of towards each other?

**Science Notebook**:

**Predict** what how youthink the flags will move as when you close and open the book.

After they have made their predictions, ask students to try it and then record the results.

**Science Notebook**:

**How** is it different from the Monster? How is it similar? What could you make with it?

## Word Bank

arc, arc length, symmetric angle-fold, far side, link angle, radius, asymmetric angle-fold, vertex

## Extensions

1. Where will the Angle-fold hide? Here is an angle-fold. Where will the flag go when the book is closed?



Figure 4: Where does the flag “hide” when the book is closed?

First, predict the answer. Then use the See-thru book to find out if your prediction was correct.

For a further challenge, predict where the Monster and Mouth will hide when the book is closed.

2. Experiments with the Monster:

1. Make the flag shorter or longer, and see how this affects the amount of motion.
2. Try making and attaching curved flags, as in Figure 5. Predict how they will move as the book opens and closes.
3. Make the flag so long it will stick out of the book, when the book is closed. Predict where it will stick out. Find the maximum length of the flag *before* it starts to stick out.



Figure 5: Monster with curved flags

3. Experiments with the Mouth:

1. Cut teeth in each jaw. Make them so the teeth mesh when the mouth closes.
2. Add a tongue. First, make a tongue that moves with one of the jaws. Then make a tongue that doesn’t move, as the jaws close around it.
3. Fit a small parallel fold between the two jaws. The jaws could be a dog’s mouth, and the parallel fold could be the bone inside.
4. Make a mouth whose top jaw moves less than the bottom jaw.
5. See what happens when you use asymmetric angle-folds for the top and bottom jaws.

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date: \_\_\_\_\_\_\_\_\_\_\_\_\_

**The Monster and the Mouth**

**1. The Monster.** Show which way the flags move as you open the book:



**2. The Mouth.** Show which way the flags move as you open the book:



3. How is the motion similar, between the Monster and the Mouth?

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

4. How is the motion different, between the Monster and the Mouth?

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

5. Explain why they move differently.

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

# Lesson 10: Peek-a-Boo and Hand-Waver

## **Essential Question**

How can you make a pop-up that combines both an angle-fold and a parallel fold, to get both kinds of motion?

## **Task**

Create and study two kinds pop-ups that use both pop-up mechanisms

## Standards:

CCLS – ELA **Writing**: Text types and purposes; Research to build and present knowledge  
**Speaking & Listening**: Comprehension and collaboration

CCLS – Math   
**Math Practices**: MP4: Model with mathematics; MP6: Attend to precision; MP7: Look for and make use of structure  
**Operations and Algebraic Thinking:** 4. OA5 & 5. OA3: Generate and analyze patterns and relationships **Measurement and Data:** 4.MD 5, 6 & 7: Understand concepts of angle and measure angles **Geometry:** 4 & 5. G1,2&3: Draw and identify lines and angles

NGSS  
**Scientific & Engineering Practices** 1. Asking questions and defining problems; 3. Planning and carrying out investigations; 6. Constructing explanations and designing solutions; 8. Obtaining and evaluating information.  
**Crosscutting Concepts:** 1. Patterns; 2. Cause and effect: mechanism and prediction; 4. Systems and system models; 6. Structure and function **Disciplinary Core Ideas:** ETS1: Engineering Design

## Outcomes

* Make two pop-ups that produce complex motion, of the kind that are found in commercial pop-up books
* Study and explain how these constructions work

## Assessment

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Objective:** | **Below (1)** | **Approaching (2)** | **Proficient (3)** | **Advanced (4)** |
| A. Create both constructions | Neither one | One but not the other | Creates both | Creates both and uses them as the basis for further experiments |
| B. Explain how both constructions work | No explanation | Explanation is incomplete | Explains each one clearly | Explains both constructions and compares them |

## Advance Preparation

* Make a Peek-a-Boo and a Hand-Waver. For guidance, view the opening video and videos under Procedure at <http://citytechnology.org/force-and-motion/the-peek-a-boo-and-the-hand-waver>.

## Materials

* Scissors, tape, rulers, ball-point pens and Post-Its ™ (or paper plus glue stick)
* Pop-ups made during Lessons 7 & 9
* Assorted pop-up books and cards, from the intermediate and advanced categories
* Peek-a-boo Template
* Hand-Waver Template
* Twister Template (for Extension #3)

## Procedure

**1. Making a peek-a-boo** (Individual – 30 min.) Demonstrate the Peek-a-Boo you have made. Provide each student with the Peek-a-boo Template. Directions are printed on the template.

**Science Notebook**:

How is the Peek-a-Boo different from the pop-ups you made before? How is it similar?

How could you use this mechanism in making your own card?

**2. How does it Work?** (Whole class / Individual – 20 min.) Lead a discussion in which students analyze this construction to see what kinds of pop-ups it’s made of, and how they are related. Some focusing questions are:

* How does it work?
* What kinds of pop-ups can you find in the Peek-a-boo? In what direction does each one move? How are they connected?
* Find this construction in commercial pop-up books. How did they use it?

Provide time for students to experiment with this construction.

**Suggested breakpoint between periods**

**3. Make a Hand-Waver** (Individual – 30 min.)Demonstrate the Hand-waver you have made.Provide each student with the Hand-waver Template. Directions are printed on the template. It is called the Hand-Waver, because some students have cut the flag in the shape of a little hand, which waves “Hello” every time you open the book.

**Science Notebook**:

How is the Hand-Waver different from the pop-ups you made before? How is it similar?

How could you use this mechanism in making your own card?

**4. How does it Work?** (Whole class / Individual – 20 min.) Lead a discussion in which students analyze this construction to see what kinds of pop-ups it’s made of, and how they are related. Some focusing questions are:

* How does it work?
* What kinds of pop-ups can you find in the Peek-a-boo? In what direction does each one move? How are they connected?
* Find this construction in commercial pop-up books. How did they use it?
* How are the Peek-a-Boo and Hand-Waver different? How are they similar?

Provide time for students to experiment with this construction.

## Extensions

1. Experiments with the Peek-a-boo:

1. Predict where the fold line will occur in the parallel fold, before you close the book. Then check the actual location to see if you were right. HINT: Where are A, B, C and D?
2. How long can you make the flag in the Peek-a-Boo? If you want to make it longer, without it bending, how can you change the design to fit a longer flag?

2. Experiments with the Hand-waver:

1. Using ideas from the commercial pop-up books, Make a two-piece Hand-waver, that uses a separate angle-fold hidden under the parallel-fold
2. Make a pop-up card that combines two Hand-wavers, one on top and one on the bottom. We call this construction the **Bird**, because the flags could be the wings, which come out as you open the book (see Figure 5).



Figure 5: The Bird

3. The Twister

The Twister is a dynamic and surprising pop-up used occasionally for special effects in commercial pop-up books. The Twister Template contains all the parts and instructions for making a Twister. Additional support is available at <http://citytechnology.org/force-and-motion/the-twister>,

Here are some focusing questions:

* How do you think the Twister works?
* What kinds of pop-ups does it use – angle-fold or parallel-folds? Where is each one? How are they attached?
* Open the book slowly, and look under the rhombus as you do so. What shapes do you see? Do they remind you of shapes you have seen before?

## 

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date: \_\_\_\_\_\_\_\_\_\_\_\_\_

**The Peek-a-boo and the Hand-waver**

**1. Direction of Motion.** Draw an arrow to show how the flag moves in each one as you open the book:



**2. Peek-a-Boo and Hand-Waver**

What makes the flag move the way it does in the Peek-a-Boo?\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

What type(s) of pop-up does the Peek-a-boo use? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

What makes the flag move the way it does in the Hand-Waver?\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

What type(s) of pop-up does the Handwaver use? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**3. Comparison**

How are the two constructions similar? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

How are the two constructions different? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Lesson 11: Make a Group Pop-up Book

## **Essential Question**

What can you make using the pop-up constructions you have learned?

## **Task**

Each group designs and makes its own pop-up book

## Standards:

CCLS – ELA **Writing**: Text types and purposes; Research to build and present knowledge  
**Speaking & Listening**: Comprehension and collaboration

NGSS  
**Scientific & Engineering Practices** 1. Asking questions and defining problems; 3. Planning and carrying out investigations; 6. Designing solutions; 8. Obtaining and evaluating information.  
**Crosscutting Concepts:** 1. Patterns; 2. Cause and effect: mechanism and prediction; 4. Systems and system models; 6. Structure and function**;   
Disciplinary Core Ideas:** ETS1: Engineering Design

## Outcomes

* A pop-up book can be used to make a story or idea come alive

## Assessment

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Objective:** | **Below (1)** | **Approaching (2)** | **Proficient (3)** | **Advanced (4)** |
| Design and make a group pop-up book | No exploration | Recognizes that the amount of motion can change, but doesn’t know how | Finds where the amount of motion is maximum | Explains why motion is maximum and uses this fact in designing a pop-up |

## Materials

* Scissors, tape, rulers, ball-point pens and Post-Its ™ (or paper plus glue stick)
* Pop-ups made during Lessons 7, 9 & 10
* Assorted pop-up books and cards, from the intermediate and advanced categories.

## Procedure

1. **Creating a Book** (Small groups – 100 min.). Divide the class into small groups. Each group is to create a pop-up book. Tell them that they will be presenting these books to an audience in Lesson 12. The book should be based on a single theme – perhaps the theme that students came up with in Lesson 1. Ideas for exciting pop-up mechanisms are available in the Extensions to Lessons 7, 9 and 10. Each member of the group should create at least one pop-up card, which will become part of the book. To create a book from separate cards, glue the cards together back to back. Encourage the groups to save all their design drawings and record both successful and failed designs, as these will be part of their presentations.

# Lesson 12: The Pop-up Show

## **Essential Question**

How well can each group present their work and respond to questions about it?

## **Task**

Present and discuss pop-up books made by each group

## Standards:

CCLS – ELA **Speaking & Listening**: Presentation of knowledge and ideas; Comprehension and collaboration **Language**: Vocabulary acquisition and use

NGSS  
**Scientific & Engineering Practices** 3. Planning and carrying out investigations; 6. Designing solutions; 8. Obtaining and evaluating information.  
**Crosscutting Concepts:** 1. Patterns; 2. Cause and effect: mechanism and prediction; 4. Systems and system models; 6. Structure and function **Disciplinary Core Ideas:** ETS1: Engineering Design

## Outcomes

* Understanding of pop-up mechanisms is enhanced through presentation and discussion

## Assessment

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Objective:** | **Below (1)** | **Approaching (2)** | **Proficient (3)** | **Advanced (4)** |
| Present and discuss group pop-up books | Present-ation is minimal or non-existent | Presentation of products, but not process | Presentation of both product and process | Engages in discussion about choices made, alternative ideas and future improvements |

## Advance Preparation

* Arrange for space and if possible an audience from outside to attend group presentations

## Materials

* Pop-up books made by groups
* Charts and tables showing design process and outcomes

## Procedure

**1. Presenting pop-up books** (Whole class – 50 min.)

* **Classroom presentation**: Each group demonstrates their pop-up book to the class. The class has to guess what it represents and how it will move when the input is operated. Students should also share the issues they encountered and what they did to solve them.
* **Bulletin board or poster display**: Pop-ups can be attached to poster boards or bulletin boards. By using push pins strategically – for example, at the corners – you can avoid interfering with the mechanism, allowing viewers to try them out to see how they work. The display should also include diagrams and other records of the design process.
* **Museum table**: For Parent-teacher Conferences, Open School Night, or other community events, the pop-ups can be displayed loose on tables with signs inviting viewers to guess what they will do and then test them.
* **Invention Convention:** Stage a science-fair style event, to give students an opportunity to explain their pop-ups to parents and other visitors.

**Glossary**

**Acute angle:** Anangle that issmaller (sharper) than a right angle

**Angle:** Amount of spreading between two intersecting lines.

**Angle-fold pop-up:** A pop-up construction that produces vertical as well as horizontal motion, and whose hinge lines intersect at one point

**Arc:** part of a circle

**Arc length:** distance traveled when something moves in an arc

**Asymmetric angle-fold:** Angle-fold with unequal page positions, and therefore different angles A and D.

**Asymmetry**: Lack of symmetry in an object.

**Attachment:** Connection between pieces makes them move together.

**Axis**: Line around which something rotates.

**Book:** The folder or outside cover of a pop-up mechanism.

**Burnish:** Use a fingernail or other hard surface to make a fold line sharp

**Cause:** What makes something else happen

**Compound system:** A system consisting of more than one subsystem.

**Controlled experiment:** An experiment designed to discover cause-and-effect relationships, by varying only one independent variable at a time

**Controlled variable**: A variable that is deliberately kept the same during a controlled experiment, so there is only one independent variable.

**Data analysis:** Process of looking for patterns in data

**Data collection:** Process of acquiring and tabulating data

**Data table:** A graphic organizer designed to present data in a way that makes it easy to find the patterns

**Dependent variable**: Variable that is expected to change as a result of changing the independent variable. This term is commonly used in science; see also output variable**.**

**Dimension:** A measure of how much motion is possible in a region; ordinary space has three dimensions, while a plane has 2 and a line has only 1.

**Direction of motion:** A path showing the way something can move

**Edge view:** What something looks like when you look at it from its edge, rather than a flat surface

**Effect:** What happens as a result of a cause

**Equation:** A symbolic statement that two quantities have the same size

**Experiment:** A method of creating new knowledge, by trying out a new idea

**Explanation:** A statement about why something happens, relating cause and effect

**Far side:** Side with the larger link angle

**Fix:** Address a problem in a design by finding its cause and removing it

**Flag:** A small piece of paper used to demonstrate motion

**Fold**: The hinge in the pop-up piece that separates it into two links

**Force component:** The amount of force in a particular direction.

**Force:** Push or pull needed to start something moving.

**Fraction:** Part of a whole

**Height**:Distance from the base.

**Hinge**: A place where either or both of two links can rotate, like a door or cabinet hinge.

**Horizontal:** Parallel to the ground

**Hypothesis:** An idea that you can test through an experiment

**Independent variable:** Variable that is deliberately changed in the course of doing an experiment, to see what effect it will have on another variable – the dependent variable. This term is commonly used in science; see also input variable**.**

**Input variable**: Variable that is deliberately changed in the course of doing an experiment, to see what effect it will have on another variable – the output variable. This term is commonly used in math; see also independent variable.

**Input:** The action you have to take to operate a system

**Intersecting lines:** Two or more lines that meet at a common point, called the vertex.Intersecting lines may intersect at any angle; if they intersect at right angles they are called perpendicular**.**

**Issue:** An aspect of a design that you would like to fix

**Length:** Distance from one point to another

**Link:** A rigid part of a mechanism, which is not supposed to bend or twist

**Linkage**: A mechanism consisting of links and hinges.

**Measurement:** Process of describing a quantity by a number

**Mechanism**: A device with moving parts.

**Obtuse angle:** Angle that is larger than a right angle

**Opaque:** Describes something you can’t see through

**Output variable:** Variable that is expected to change as a result of changing the input variable. This term is commonly used in math; see also **dependent variable.**

**Output:** The result of applying the input to a system

**Page positions**: The two places, one on each side of the gutter, where the pop-up is attached to the book. These are both hinges, because they allow rotation of links

**Pages**: The two sides of the book, separated by the gutter.

**Paper engineer:** A person who designs pop-ups as a profession

**Parallel connection:** System arrangement in which two subsystems have the same input

**Parallel lines:** Two or more lines that run in the same direction without meeting, and are always the same distance apart.

**Parallel-fold pop-up:** A pop-up construction that cannot produce up-and-down motion, and whose hinge lines run parallel to each other

**Pattern:** Repeating set of elements

**Perpendicular:** Describes lines that are at right angles to each other. Perpendicular lines are a special case of intersecting lines

**Plane:** A two-dimensional surface, such as the top of a sheet of paper

**Point of view:** Your position in relation to an object you are looking at

**Pop-up piece**: The inside piece of a pop-up mechanism.

**Quadrilateral:** Any four-sided shape

**Radius:** distance of an arc or circle from the axis

**Rectangle:** Rhombus with all four angles = 90○

**Rhombus:** Quadrilateral with opposite sides parallel

**Right angle:** An angle with two perpendicular sides, like the corner of a square

**Score:** Make a straight groove in a piece of paper by pressing it with a pointed object

**Series connection:** System arrangement in which the output of one subsystem is the input to another

**Skew lines:** Describes lines that are neither intersecting nor parallel

**Space:** Three-dimensional region

**Square:** Rectangle with all four sides equal

**Subsystem:** Part of a system that could be a system in itself

**Symbolic representation:** Description that uses agreed-upon symbols rather than words

**Symmetric angle-fold:** Angle-fold with equal page positions, and therefore the same angles A and D

**Symmetry:** A pattern that occurs more than once, for example as a mirror image.

**System Diagram:** Block diagram showing relationships among inputs, outputs and subsystems

**System of simultaneous equations:** Mathematical expression of multipleconditions thatneed to occur together

**System:** A collection of parts that function together as a whole, operated by an input and resulting in an output

**Test:** Find out whether or not a hypothesis is true

**Three-dimensional:** Characteristic of space

**Top view:** What something looks like when you look down at it

**Transparent:** Describes something you can see through

**Triangle:** Three-sided figure

**Troubleshooting:** Identifying an issue and its cause before trying to fix it

**Two-dimensional:** Restricted to a plane

**Variable**:A property of an object or system that can change in the course of doing an experiment.

**Vertex:** The point where intersecting lines meet.

**Vertical**: Up and down